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Stereoscopic matching and the aperture problem

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Abstract. In order to perceive stereoscopic depth, the visual system must define binocular disparities. Consider an oblique line seen through an aperture formed by flanking occluders. Because the line is perceived behind the aperture, the line must have disparity relative to the aperture. What is the assigned disparity of the line in this aperture problem? To answer this question five observers adjusted the horizontal disparity of a probe until it was perceived at the same depth as the disparate line behind the aperture. The results show that, when both the horizontal and the vertical disparities of the occluders are well-defined, the probe must have the same horizontal disparity as the horizontal separation between the line half-images. However, when the horizontal and vertical disparities of the occluders are ill-defined, the intersections of the line and the occluder borders can determine the matching direction. In the latter case, the matching direction varies with the aperture orientation and there is considerable variability across observers.

1 Introduction

The stereogram in figure 1a shows a horizontally shifted line and a zero-disparity occluder.[1] When the two stereogram half-images are being fused, the line clearly appears behind the occluder and all parts of the line have the same depth. In figure 1b, the two half-images are superimposed, showing that the horizontal shift of the line results in shifts in several directions across the straight apertures.

Is the line in figure 1 binocularly matched according to these aperture-induced shifts, or is there a single matching direction for all these line parts? More generally, what is the assigned disparity of a line that is observed through an aperture? To investigate this problem it is useful to concentrate on conditions in which matching is unconstrained.[2] Note that in figure 1 the endpoints of the test line are visible and these points might constrain the match for the whole line by interpolation (Mitchison and McKee 1987; van Ee and Schor 2000). Figure 2 shows a stereogram similar to that of figure 1, but now the endpoints of the test line are no longer visible. The line has a well-defined non-zero horizontal shift relative to the occluders. When the half-images are fused, the test line appears behind the occluders. Considering that the test line could theoretically have its endpoints everywhere behind the occluder, matching of the line is highly ambiguous. This means that a point on one half-image of the line could theoretically be matched with every point of the other eye's half-image.

Although there are many studies on the aperture problem for stereopsis and also many binocular-matching studies, binocular matching for partially occluded objects has

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[1] For consistency, disparity is specified in screen coordinates and not in retinal coordinates.

[2] The horizontal and vertical tolerance to disparity for binocular fusion was described by Tyler (1975) and their dynamics by Schor and Tyler (1981). These studies were primarily concerned with disparity limits for fusion and used constrained matching features. We are primarily interested in the stimulus locations that become matched from the multitude of possible locations that can be fused in unconstrained matching stimuli.

(a)

(b)

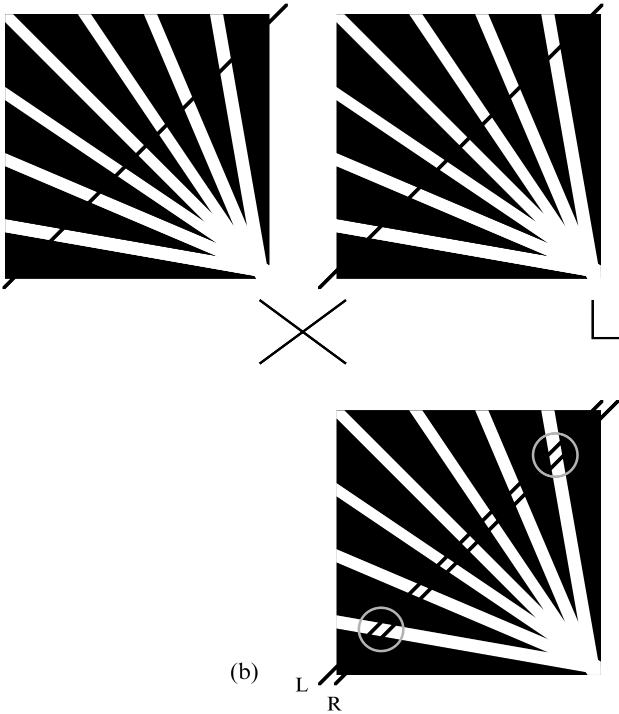

Figure 1. (a) Demonstration of the aperture problem for stereopsis. Crossfusers should fuse the centre and the left image. Divergers should fuse the centre and the right image. (b) Superposition of the half-images of the stereogram. The shift of the half-images of the line changes with aperture orientation from nearly horizontal (grey circle bottom left) to nearly vertical (grey circle top right).

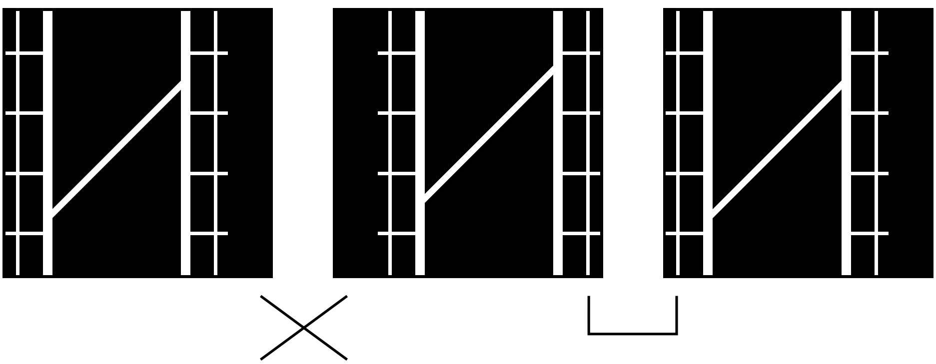

Figure 2. Stereogram depicting an oblique line seen through a vertical aperture formed by flanking occluders.

not been quantitatively investigated in any of them.[3] For such an aperture problem there is a continuum of matching solutions. The literature provides at least four plausible hypotheses, the predictions of which are indicated in figure 3. This figure shows the superposition of the half-images for the stereogram of figure 2. The four matching predictions are labeled the horizontal match (arrow *1*), the perpendicular match (arrow *2*), the intersection match (arrow *3*), and the aperture-direction match (arrow *4*). These predictions will be described below.

[3] Existing studies on stereoscopic matching of gratings or similar structures with apertures and occlusions (Tyler 1980; Morgan and Castet 1997; van Ee and Anderson 2001; van Ee et al 2001) used stimuli that are fundamentally different from ours.

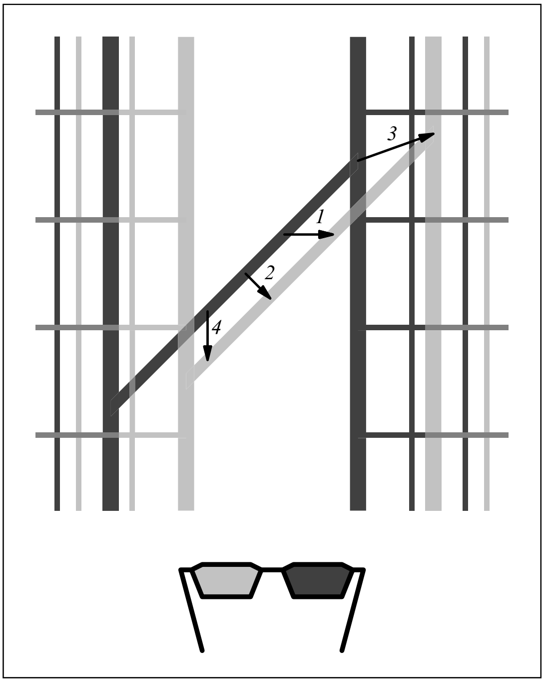

Figure 3. Superposition of the stereogram half-images of figure 2. There are at least four theoretically plausible matching directions: a horizontal match (*1*); a match perpendicular to the orientation of the test line (*2*); a match according to the visible termination points of the test line—we call it the intersection match (*3*); and the aperture-direction match (*4*).

1.1 *Horizontal match*

The horizontal match refers to a match in the horizontal direction (arrow *1* in figure 3). Since the eyes are separated horizontally, and since nonhorizontal directions do not match corresponding portions of objects from an epipolar geometrical point of view (Anderson 1999a), horizontal-matching theories have dominated the studies on binocular vision. However, there is also undebated experimental evidence that matches can occur in other directions (eg Stevenson and Schor 1997; van Ee and Schor 2000). In a study that resembles the current study, van Ee and Schor found that, for a long line (unconstrained matching), matching occurs according to a default match, which has a vertical component as well as a horizontal component.[(4)] Even though this default match is not strictly horizontal, it is very close to horizontal and in many studies horizontal-matching models apply very well. Especially in computer vision, matching computations are immensely simplified by assuming a horizontal-matching direction (epipolar geometry).

In several studies it has been suggested that horizontal matching of the half-images occurs, and that components of the half-images that are left unpaired signal to the brain that there must be an occlusion (Nakayama and Shimojo 1990; Anderson 1994; Anderson and Nakayama 1994; Malik et al 1999).

1.2 *Perpendicular match*

The perpendicular match refers to a match perpendicular to the orientation of the test line as indicated by arrow *2* in figure 3. Note that the perpendicular matching direction is equivalent to nearest-neighbour matching.

Farell (1998) studied the perceived depth evoked by stereoplaids (figure 4). To create the plaids, he superimposed two sinusoidal gratings with different orientations and different horizontal shifts between the two half-images. He reasoned that these different

[(4)] This vertical range is approximately 1/6 of the horizontal disparity range that can be used to process static stereoscopic depth (Schor et al 1984) and 1/24 of the horizontal disparity range for dynamic stereoscopic depth (Richards and Kaye 1974).

shifts ought to result in different perceived depths for the two gratings. However, he observed that these two superimposed gratings are perceived in one single depth plane and not in two depth planes. See figure 4c for a demonstration of his effect. Note that Farell presented his stimuli for short observation periods. With prolonged viewing, the reader might (just like some of our observers did) perceive both one single plaid and distinct depth of the component gratings. Farell used an adaptation method and suggested that, although 2-D features (intersections) seem to determine the perceived depth polarity (see intersection-match prediction below), the 1-D components (gratings) serve as primitives for perpendicular matching on the mechanistic (disparity-detector) level.

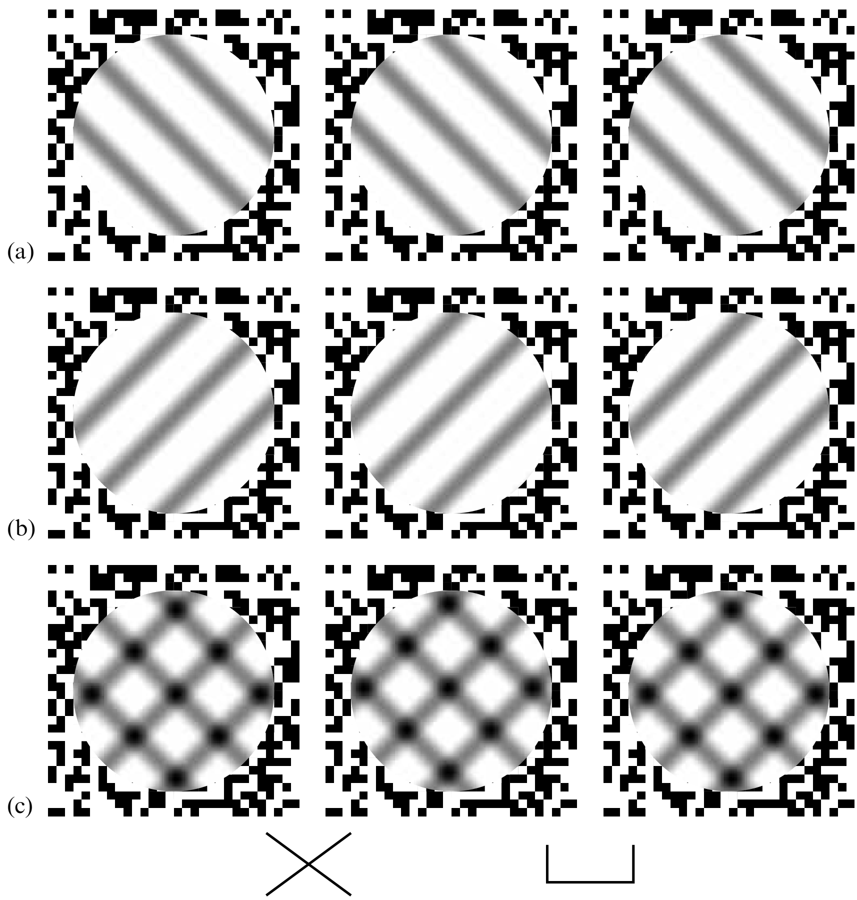

Figure 4. Example of partly occluded stereoplaids composed of sinusoidal gratings. Crossfusers should fuse the centre and the left images; divergers should fuse the centre and the right images. The random-dot pattern constitutes a zero-disparity background. (a) A grating with an orientation of 45° to vertical. There is no horizontal shift between the two half-images, so that the grating is perceived at the same depth as the random-dot pattern; (b) a grating with an orientation of −45° to vertical. The half-images have uncrossed disparity and the grating is perceived behind the random-dot pattern; (c) the stereograms of (a) and (b) are superimposed. When fused the two gratings can be perceived at the same depth to form one single stereoplaid. In other words, in accordance with Farell (1998), one does not necessarily perceive the individual components at their distinct depths as given in (a) and (b).

1.3 *Intersection match*

We refer to the intersection match as the match for which the visible termination points of the test line are being matched (arrow *3* in figure 3). These termination points reside in the half-images where the test line 'intersects' the border of the occluder.

Stereoplaids are examples in which such intersection points seem to determine the perceived depth polarity (see figure 4c). Farell (1998, page 690) stated: "It is clear that relative depth is not predictable from component disparities alone". And he continued: "What does predict the depth judgements" of these plaids "is the horizontal component of the disparity of the plaids' 2-D features". According to this hypothesis, the perceived depth polarity of the plaid (the superimposed gratings) corresponds to the sign of the horizontal disparity of the intersections of the two gratings.

Another possible argument for the intersection match is that the visual system can interpret the test line within the aperture as an object which is completely visible and not as a longer line of which only a part can be seen. In this case, the visible end-points (ie the intersection points of the test line and the occluder) would determine the matching direction so that every visible point of the test line in one half-image is matched with a visible point in the other eye's half-image (leaving no unpaired regions).

1.4 *Aperture-direction match*

In our stimulus of figure 2 and figure 3, the main axis of the aperture is oriented vertically. The aperture-direction match (arrow *4* in figure 3) implies that the test-line half-images are matched in the direction of the main axis of the aperture (vertical in our example). This would mean that the matching direction is determined by the aperture orientation. The aperture-direction prediction has a well-known analogue in the aperture problem for motion, namely the barber-pole effect. In the barber-pole effect a line is moving behind an aperture. If the motion of the line has a component perpendicular to the line orientation, the line will appear to be moving along the main axis of the aperture, regardless of the true motion of the line. Because of the many similarities between the aperture problem for motion and the aperture problem for stereopsis (Shimojo et al 1988; Anderson 1994; Anderson and Sinha 1997), it might be the case that the aperture direction influences binocular matching.

2 General methods

To examine the assigned disparity (the matching direction) of a line that is observed through an aperture we used a depth-probe method (Richards 1971; Mitchison and McKee 1987; van Ee and Schor 2000). In this method, a probe is shown besides the test line as in figure 5. The test-line half-images, the dark-grey and light-grey lines in figure 5, have a certain disparity and the line is thus perceived in a certain depth plane. The right eye's half-image of the probe has a fixed position on the screen, but the position of the left eye's half-image of the probe can be varied along a horizontal line. In this way the horizontal disparity, and thus the perceived depth of the probe, is varied (van Ee and Schor 2000). When the probe and the line are perceived at the same depth, the geometrically assigned horizontal disparity for the probe and the line ought to be the same.[5] Since the horizontal disparity for the probe is unambiguous, the assigned horizontal disparity of the line (ie the horizontal disparity component of the match) can be measured in this way.[6]

[5] Van Ee and Schor (2000) tested this assumption by using short test lines and adding vertical disparities to the probe. We replicated their results in our setup, before conducting the experiments described in this paper.

[6] While the depth-probe analysis is correct from a geometrical point of view, it is not necessarily a measure for the matching direction on disparity-detector level. The depth-probe analysis reveals the matching direction for the assigned horizontal disparity (see also footnote 5). Farell (1998, 1999, 2003) refers to matching in terms of the underlying disparity detectors, meaning that there can be a difference between the horizontal disparity that is assigned (high-level) to the stimulus and the mechanistic (low-level) matching direction. Concerning this issue, it is relevant to cite a recent debate in the literature between Farell and Anderson (Farell 1998, 1999; Anderson 1999a) on the definition of disparity. According to Anderson (1999a), whose work concerns the high-level disparity assignment, matching is horizontal since nonhorizontal directions do not match corresponding portions of objects.

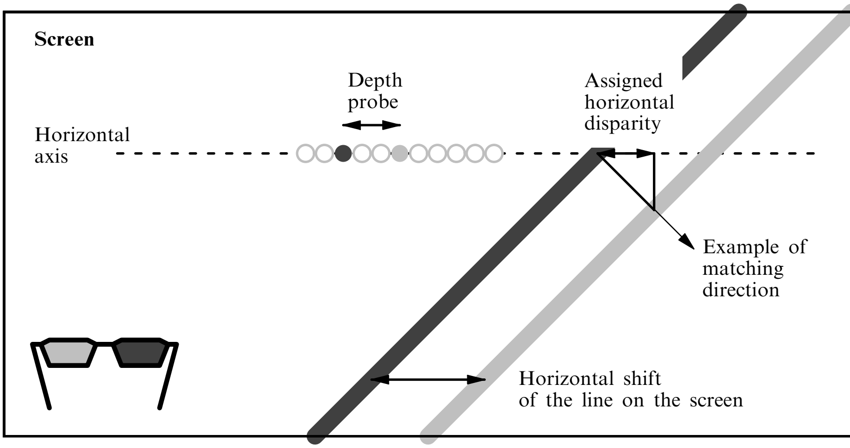

Figure 5. The depth-probe method. The dark-grey line and dot are seen by the right eye and the light-grey line and dot are seen by the left eye (crossed disparities). The horizontal disparity of the probe is varied. By asking the subjects whether they see the probe in front or behind the test line, the assigned horizontal disparity of the test line can be obtained. In this figure the geometrically assigned horizontal disparity corresponds to a matching direction which is perpendicular to the test line.

2.1 *Apparatus*

The anaglyphic stimuli were projected onto a large screen (240 cm by 190 cm) in an otherwise dark room with black walls, ceiling, and floor. The intensities of the red and green stereogram half-images were adjusted until they appeared equally bright when viewed through the red and green filters, respectively. There was no visible crosstalk between the half-images. One pixel subtended 3 min of arc by 3 min of arc and lines were anti-aliased. The image on the screen was refreshed every 14 ms. To obtain a fixed head position and orientation, a chin-rest was used. This chin-rest was positioned 200 cm from the screen.

2.2 *Stimuli*

Several different stimulus patterns were used in the experiments (see figures 6, 7, and 8). A fixation cross (0.9 deg by 0.9 deg) was presented in the centre of the screen.

Two types of occluder conditions were examined in this study (see figure 8a). For the first type, the occluders consisted of grids on either side of an aperture. The occluders were placed so that the fixation cross was in the middle of the aperture. The size of a rectangle in the grids was 1.3 deg by 4.3 deg and the size of the entire stimulus was 44 deg by 40 deg. 5% of the rectangles in the occluders were filled to prevent subjects from fusing the grid in the wrong depth plane (the wallpaper effect). The width of the lines within the grid was 0.26 deg. At the borders of the aperture, thicker lines were drawn with a width of 0.4 deg. The horizontal disparity of the occluders was 26 min of arc relative to the fixation cross. For consistency, we will refer to these occluders as surface occluders throughout this paper.

For the second type of occluder conditions, the occluders consisted of single lines specifying only the borders of the occluder. The length of these lines was 40 deg and the width was 0.4 deg. This line-occluder condition is interesting because it provides a stimulus in which there is no clear vertical disparity information for the visual system, as opposed to the surface-occluder condition for which global vertical disparity is well defined. It could be that, when there is no clear matching information for the occluder itself, the intersections of the test line with the occluder borders become more important.

The orientation of the test line within the aperture was 45° to horizontal. The width of the test line was 0.5 deg and its length was 3 deg. This length was chosen such that the test line fitted the width of the aperture. The horizontal separation between the test-line half-images was 17 min of arc.

The depth probe was always presented simultaneously with the test line. The diameter of the depth probe was 0.5 deg. The depth probe was presented slightly above the position of the fixation cross and the test line was presented slightly below. The distance between the fixation cross and both the depth probe and the test line was 1.5 deg.

2.3 *Task and procedure*

At the beginning of every session, both the stereoscopic surface occluders and the fixation cross were presented for 11.5 s. The grid structure of the surface occluders helped to stabilise vertical and cyclotorsional eye alignment (van Ee and Schor 2000; van Ee and van Dam 2003). At the start of each trial, the surface occluder and fixation cross were visible for 1.5 s (see figure 6). Subsequently, the fixation cross disappeared for 98 ms to prevent masking effects. In this period, the surface occluders remained on the screen for trials in which the surface occluders were present (figure 6a). For other trials the screen was completely black (figure 6b). Then the oblique test line and the depth probe were flashed within the aperture for 196 ms.[7] Afterwards, either the grid was shown or the screen became black for 98 ms, depending on the condition, until the fixation cross (and surface occluders if the screen was black) appeared again.[8] Subjects then pressed a button to indicate whether they had perceived the depth probe in front or behind the test line. After the subject had responded, the surface occluders and fixation cross remained visible on the screen for 1.5 s before the next trial started.

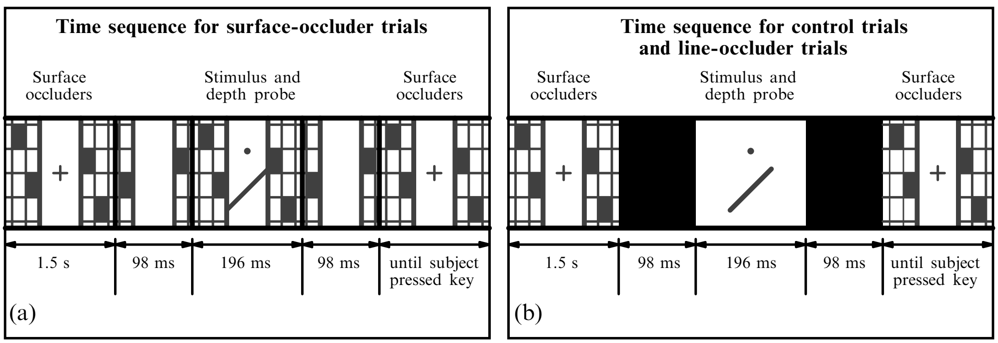

Figure 6. (a) Procedure for surface-occluder trials. (b) Procedure for line-occluder trials and control trials. For the line-occluder condition (not shown) only the borders of the occluders were defined by single lines. In the control trials (shown) no occluders were present.

A 1-up/1-down staircase method was used to determine at which horizontal disparity subjects perceived the depth probe at the same depth as the test line. For the different conditions, the staircases were interleaved. Step size was initially 2 pixels (6.0 min of arc), but was reduced to 1 pixel (3.0 min of arc) after the second reversal and to 0.5 pixel (1.5 min of arc) after the fourth reversal. This subpixel step size of the probe

(7) Note that this time is sufficiently short to prevent eye movements. Saccades can be planned and started but not completed, and should therefore not interfere.

(8) Before conducting the experiments, we tested different exposure times (0 and 50 ms blank time between fixation and stimulus, and stimulus presentations of 126 and 154 ms). We found that exposure time did not significantly affect the results. However, with shorter presentation and blank times the task became increasingly more difficult to perform, and staircases frequently did not converge within 60 stimulus presentations.

could be obtained by conventional anti-aliasing techniques. The result from a single staircase measurement was obtained from the last 12 reversal points. For every condition, there were two such staircase measurements.

In order to determine biases in the responses of the subjects, there was a control condition, in every experimental session. For this condition no occluder was present and the length of the test line was 0.5 deg. This line length was chosen such that the test line had about the same size as the depth probe. In this condition, the test-line half-images were shifted horizontally by 17 min of arc. Because of its short length, matching of this test line is unambiguous and the results for this condition can be used as a measure for the bias in the depth-probe settings (van Ee and Schor 2000). Results for other conditions were corrected afterwards for this bias.

Five subjects participated in the experiments. All subjects had normal or corrected-to-normal vision. The subjects were tested with a stereo-anomaly test (van Ee and Richards 2002) and all were able to distinguish disparities of different signs and magnitudes within a range of -1 to 1 deg.

3 Experiment 1: Vertical aperture

In the first experiment we used a vertical aperture. The width of this aperture was 2.1 deg so that the length of the oblique test line within the aperture was 3 deg. We examined both the surface-occluder condition and the line-occluder condition. For the intersection match, the test line ought to have the same assigned horizontal disparity component as the vertical occluder (26 min of arc). For the horizontal match, the assigned horizontal disparity ought to be the same as the horizontal separation of the test-line half-images (17 min of arc). For the perpendicular match, the assigned horizontal disparity ought to be half the horizontal separation (8.5 min of arc), since the orientation of the test line was 45° to horizontal. For the aperture-direction match (in this case vertical), the assigned horizontal disparity ought to be 0 min of arc.

There were three control conditions in this experiment. In these conditions, the test line had the same length (3 deg) and position as the test lines in the occluder conditions, but there was no occluder present in the stimulus (see also figure 6). The test-line half-images could be shifted either horizontally, vertically, or in a way as if an occluder would have been present ('gapwise shift'). For the latter condition, the gapwise shift, the endpoints in the half-images of the test line were at the same positions as the intersections for the occluder condition would have been. The horizontal separation between the half-images of the test line was always 17 min of arc.

Van Ee and Schor (2000) showed that with increasing line length the visual system matches the test line more and more according to a default match, which is nearly horizontal. This means that if the test line continues into the periphery, the default match is predicted, whether or not the occluders are actually there. Thus, to study the matching direction of the occluded test line, the length of the test line (and therefore the width of the aperture) is limited. To check whether the test-line length of 3 deg exceeds this limit, we first checked whether the test line can still be matched according to endpoint matching (when occluders are absent). If so, then the results for the gapwise-shift condition should be near the prediction for the intersection match; the results for the horizontal-shift condition should be near the prediction for the horizontal match; and the results for the vertical-shift condition should be near the aperture-direction match prediction.

We estimated horizontal and vertical fixation disparities using a nonius-line procedure described in van Ee et al (1999). The staircases for the nonius condition were intermixed with the stimuli staircases. In this way the fixation disparity could be directly related to the outcomes of the stimulus staircases.

3.1 *Results*

We first present the results for the control conditions, since these also justify the setup for this experiment (figure 7). The results for the gapwise-shift condition are presented in figure 7a, the results for the horizontal-shift condition are presented in figure 7b, and the results for the vertical-shift condition are presented in figure 7c. Each of the panels of figure 7 depicts the horizontal disparity of the probe, for which the probe was perceived in the same depth plane as the test line. The mean across all five subjects is presented by the sixth bar in each panel. The error bars represent standard errors.

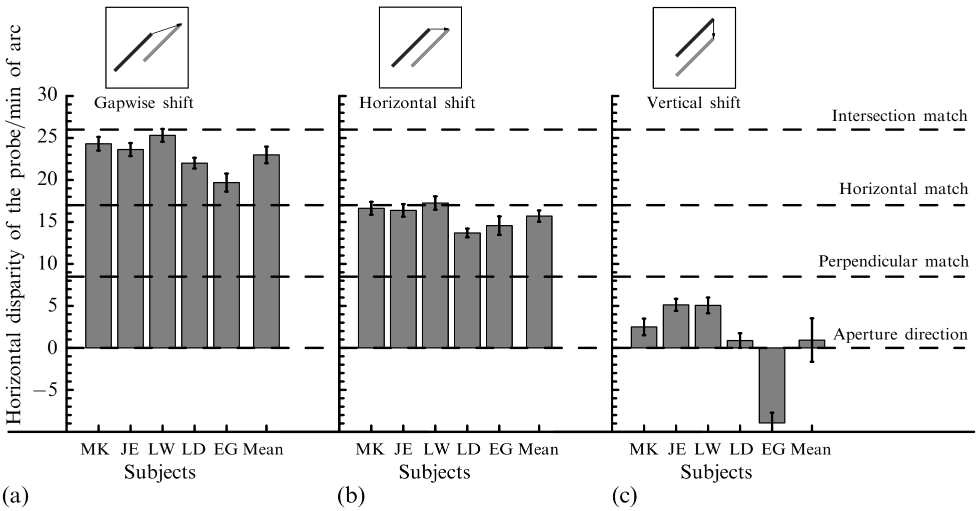

Figure 7. Results for the control conditions in experiment 1. The horizontal disparity of the probe, for which the probe and the test line are perceived at the same depth, is plotted for every subject. The dashed lines indicate the four theoretical predictions: (a) the results for the gapwise shift—for this condition the endpoints of the test line had a horizontal disparity component of 26 min of arc, which corresponds to the intersection-match prediction; (b) the results for the horizontal shift—for this condition the endpoints of the test line had a horizontal disparity of 17 min of arc, which corresponds to the horizontal-match prediction; (c) results for the vertical shift—for this condition, the endpoints of the test line had a horizontal disparity component of 0 min of arc, which corresponds to the aperture-direction prediction. Error bars represent standard errors. The results indicate that the endpoints of the test line were being matched.

The results for the control conditions show that the endpoints of the test line were being matched as predicted. The results for the gapwise shift are close to the prediction for the intersection match. The results for the horizontal shift are close to the horizontal-match prediction. The results for the vertical-shift condition are somewhat diverse but overall they are close to the aperture-direction (vertical) prediction. This also means that the intersections of the test line with the occluder borders are not located too far in the periphery to be used for matching. This in turn justifies the choice of the test-line length of 3 deg for the occluder conditions.

For the occluder conditions, the horizontal disparity of the probe for which the probe was perceived at the same depth as the test line is presented in figure 8a for the surface-occluder condition and in figure 8b for the line-occluder condition. The mean across all subjects is again presented by the sixth bar in the figure for each condition. The dashed lines indicate the predictions for the four matching hypotheses. Error bars represent standard errors. The mean for the surface-occluder as well as the mean for the line-occluder condition indicate that the test line in these conditions was matched in the horizontal direction.

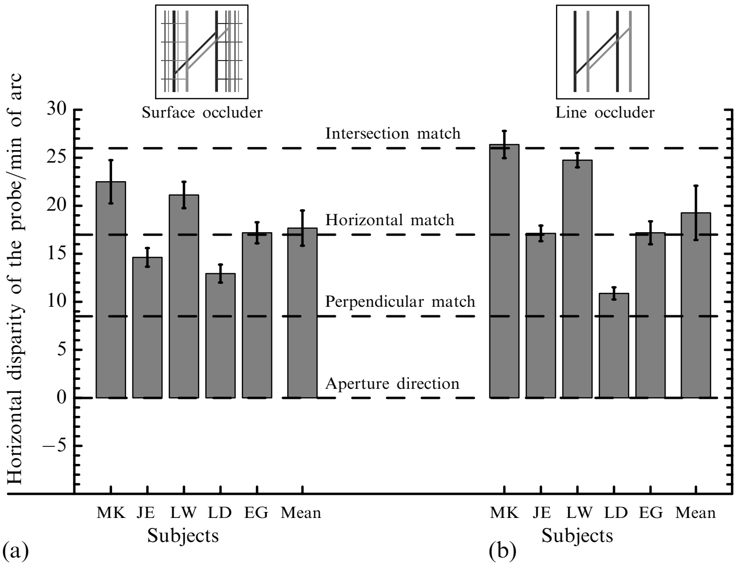

Figure 8. Results for the occluder conditions in experiment 1. The horizontal disparity of the probe for which the probe and the test line are perceived at the same depth is presented for every subject. The dashed lines indicate the theoretical predictions. For the intersection match the test line ought to have the same horizontal disparity component as the occluder (26 min of arc). For the horizontal match the horizontal disparity ought to be the same as the horizontal separation of the test-line half-images (17 min of arc). For the perpendicular match the horizontal disparity ought to be 8.5 min of arc (half the horizontal separation of the test-line half-images). For the aperture-direction match (in this case vertical) the horizontal disparity ought to be 0 min of arc. (a) The results for the surface occluder; (b) the results for the line occluder. Error bars represent standard errors. For the surface-occluder condition the results indicate that matching took place roughly in the horizontal direction. For the line-occluder condition the matching direction is subject-dependent.

For the surface-occluder condition the results for individual subjects are centred more-or-less around the horizontal-match prediction, although the result for subject MK is closer to the intersection-match prediction and the result for subject LD is inbetween the horizontal-match and the perpendicular-match predictions. Although there are relatively large differences between the subjects, it is clear that none of the subjects matched the test-line half-images in the aperture direction. The results shown in figure 8 differ significantly from both the aperture-direction prediction and the results for the vertical-shift control condition (figure 7c).

For the more ambiguous line-occluder condition, there are strong differences in matching directions between the subjects. The results for subjects MK and LW indicate that they matched the intersection points of the test line; subjects JE and EG matched the test line in the horizontal direction; and the result for subject LD corresponds more-or-less to the perpendicular-match prediction [although this could be due to underestimation which is also present in the results for the control conditions for this subject (figures 7a and 7b)].

3.2 *Fixation disparity*

We used a large (crossed) disparity of the surface occluders relative to the fixation cross, because some subjects had difficulties in matching uncrossed test-line disparities. Such a large disparity might have evoked fixation disparity. Horizontal fixation disparity does not have an effect for the horizontal match or the intersection match, but it affects both the perpendicular match and the aperture-direction match. As noted, during the experiment we estimated horizontal fixation and vertical fixation disparity.

We did not find significant vertical fixation errors. And the pattern of horizontal fixation disparity was completely unrelated to the pattern of results in figure 8.

For those subjects who did not have difficulties in matching uncrossed test-line disparities we repeated experiment 1. The occluders had zero disparity relative to the fixation cross and the uncrossed disparity of the test line was 17 min of arc. The procedure and task were the same as in experiment 1 and the session also contained nonius trials to measure fixation disparity. Figure 9 shows the results along with the theoretical predictions. The results are similar to those in experiment 1 and support our suggestion that the test line was matched horizontally. Again, the fixation errors of the subjects were unrelated to their matching results.

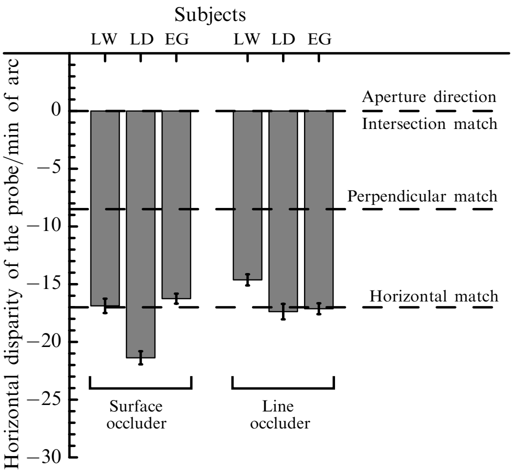

Figure 9. Results for the occluder conditions when the aperture disparity was zero relative to the fixation cross. The horizontal disparity of the probe for which the probe and the test line are perceived at the same depth is presented for subjects LW, LD, and EG. The black dashed lines indicate the theoretical predictions. Error bars represent standard errors. The results show that matching occurred in a roughly horizontal direction.

4 Experiment 2: Various aperture orientations

The aperture orientation affects the above-mentioned matching predictions in fundamentally different ways. Using the aperture orientation as a variable enables us to examine to what extent the theoretical predictions are valid.[9] It also enables us to examine the puzzling differences in matching results across subjects that we found in experiment 1.

4.1 *Stimuli and procedure*

For this experiment the setup was the same as in the first experiment except that the main axis of the aperture could now have various orientations of $-70°$, $-20°$, $0°$, $20°$, $45°$, $70°$, or $90°$ to vertical (positive angles meaning anticlockwise variation from vertical). The orientation of the test line was always $45°$ to the horizontal (which is $-45°$ to the vertical). In order to maintain a fixed line length of 3 deg, the width of the aperture varied with aperture orientation.[10]

[9] There are fundamental reasons why we chose to manipulate aperture orientation instead of test-line orientation. For instance, when the test line is more horizontal, one would expect a horizontal match to become more noisy (van Ee and Schor 2000) owing to increasing ambiguity for that match. Similarly, for a more vertical test line, one would expect less ambiguity for a horizontal match but the predictions for the intersection match and the horizontal match would be closer to each other (most of the variation being in the vertical component of the intersection match), making it only harder to distinguish between the two predictions.

[10] For the vertical-aperture condition we conducted pilot experiments for several aperture widths (test-line length thus changing accordingly). We did this for the occluder conditions as well as the gapwise-shift condition. For the vertical-occluder condition the results did not change significantly with aperture width. However, with increasing width the results for the gapwise shift converged to a more-or-less horizontal-matching direction (van Ee and Schor 2000) indicating that the endpoints in this case, and thus possibly the intersection points in the occluder conditions, were too far in the periphery to be of any use for the binocular-matching system.

In this experiment the occluders consisted again of either the surface occluders or the single lines defining the borders (line occluders). For the different orientations of the aperture, the horizontal and vertical lines of which the surface occluders consisted did not change orientation (see also figure 10). The only lines that changed their orientation were the lines defining the border of the surface occluders. The horizontal shift of the occluders was again 26 min of arc in all aperture-orientation conditions. To prevent masking effects, the pattern that was on the screen between trials (fixation cross and surface occluders) contained the aperture orientation for the next trial.

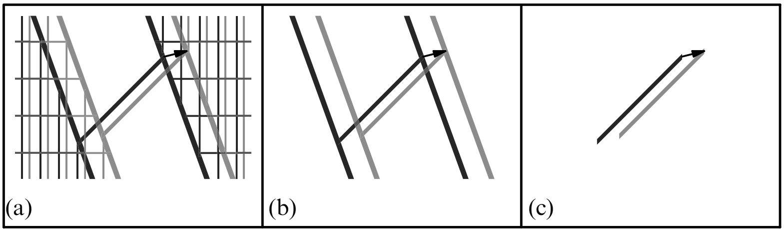

Figure 10. Stimuli in experiment 2 in which the assigned horizontal disparity of the test line with varying aperture orientations was measured. Dark-grey lines are seen by the right eye, light-grey lines by the left eye. (a) The surface-occluder condition; (b) the line-occluder condition (only the borders of the occluders are specified); (c) the gapwise-shift condition where the test line was shifted in the same way as for the occluder conditions (as indicated by the arrows), but with no occluder present in the stimulus. For the gapwise-shift condition these visible endpoints of the test-line half-images ought to be matched so that the results for this condition ought to correspond to the intersection-match prediction.

As a control, the gapwise-shift condition was examined for the various aperture orientations. For this condition, there was no occluder present in the stimulus, but the endpoints of the test line were located at the same positions as the intersection points in the occluder conditions would have been. In this case, the visible endpoints of the test line ought to be matched (van Ee and Schor 2000; see also the results of experiment 1). Thus, if for the occluder conditions subjects match the intersections points when viewing the stimulus, the results for these conditions ought to be the same as those for the gapwise-shift condition.

4.2 *Predictions*

If the test line is matched horizontally, then the aperture orientation should have no influence on the assigned horizontal disparity of the line. Accordingly, the disparity of the probe should be close to the horizontal separation of the test-line half-images on the screen (17 min of arc), when perceived at the same depth as the test line. However, if the test line is matched by using the intersections with the occluder, then the assigned horizontal disparity should vary with aperture orientation. How this horizontal disparity varies with aperture orientation is derived below (see also figure 11).

Let the intersection of the test line and the occluder border for the right eye be the origin (0, 0). Then if α is the angle of the test line to horizontal and dl is the horizontal separation of the test-line half-images, the test line in the left eye can be described as follows.

$$\text{Test line left eye} = \begin{pmatrix} \mathrm{d}l \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ \tan\alpha \end{pmatrix}. \qquad (1)$$

In this equation, λ is a variable. If β is the angle of the aperture to vertical and da is the horizontal disparity of the occluder, then the occluder border in the left eye can be described as:

$$\text{Occluder border in the left eye} = \begin{pmatrix} \mathrm{d}a \\ 0 \end{pmatrix} + \gamma \begin{pmatrix} -\tan\beta \\ 1 \end{pmatrix}. \qquad (2)$$

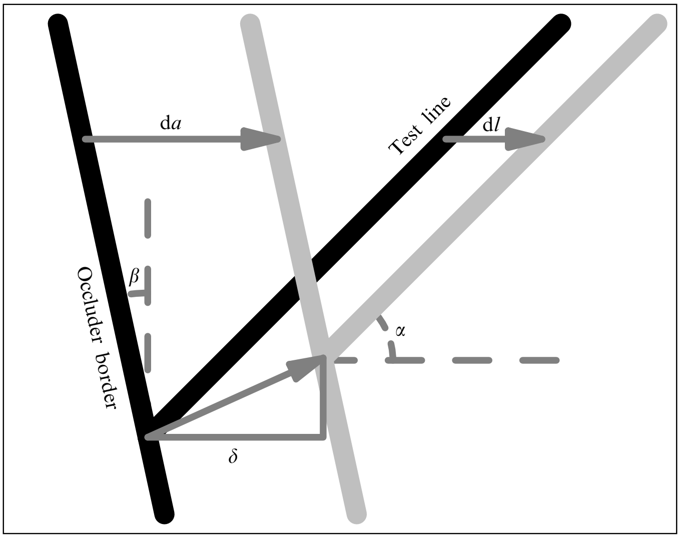

Figure 11. Schematic picture indicating the parameters that play a role in the derivation of the horizontal-disparity component for the intersection match. Dark-grey lines are seen by the right eye, light-grey lines by the left eye; dl is the horizontal separation between the test-line half-images on the screen; da is the horizontal disparity of the occluders (on the screen); α is the angle of the test line to horizontal. β is the orientation of the aperture to vertical. δ is the assigned horizontal disparity component in the case of the intersection match.

Here, γ is a variable. When these lines intersect, it follows from these equations that $\gamma = \lambda \tan\alpha$ and $\lambda = (\mathrm{d}a - \mathrm{d}l)/(1 + \tan\alpha\tan\beta)$. The horizontal component δ of this intersection point is then given by:

$$\delta = \mathrm{d}l + \frac{\mathrm{d}a - \mathrm{d}l}{1 + \tan\alpha\tan\beta}\,. \tag{3}$$

This is also the horizontal component of the disparity between the intersection point in the right and left eye, and thus the horizontal disparity of the intersection match.

4.3 *Results*

The results of experiment 2 are shown in figure 12. The filled circles represent the results for the surface-occluder condition. The open circles represent the results for the line-occluder condition and the squares represent the results for the gapwise-shift condition. The black dashed lines show the prediction for the horizontal match. The grey dashed lines represent the prediction for the intersection match. The error bars represent standard errors. The singularity in the prediction for the intersection match (at $-45°$) arise because the aperture has the same orientation as the test line and thus they never intersect. The significance within a subject was tested pairwise for each combination of data points, with a two-sided mean-difference test with a normal distribution as the underlying distribution.

The main result of this experiment is that, when subjects viewed the surface occluder (black discs in figure 12), they mainly matched the line horizontally. The results for the surface-occluder condition are close to the horizontal-match prediction, although some subjects show an overall overestimation and some others show an overall underestimation compared to this prediction.

The results for the gapwise-shift condition (white squares in figure 12) are close to the intersection-match prediction for all subjects. [For a number of subjects (eg LD) there is, however, an overall underestimation. This underestimation was also present in

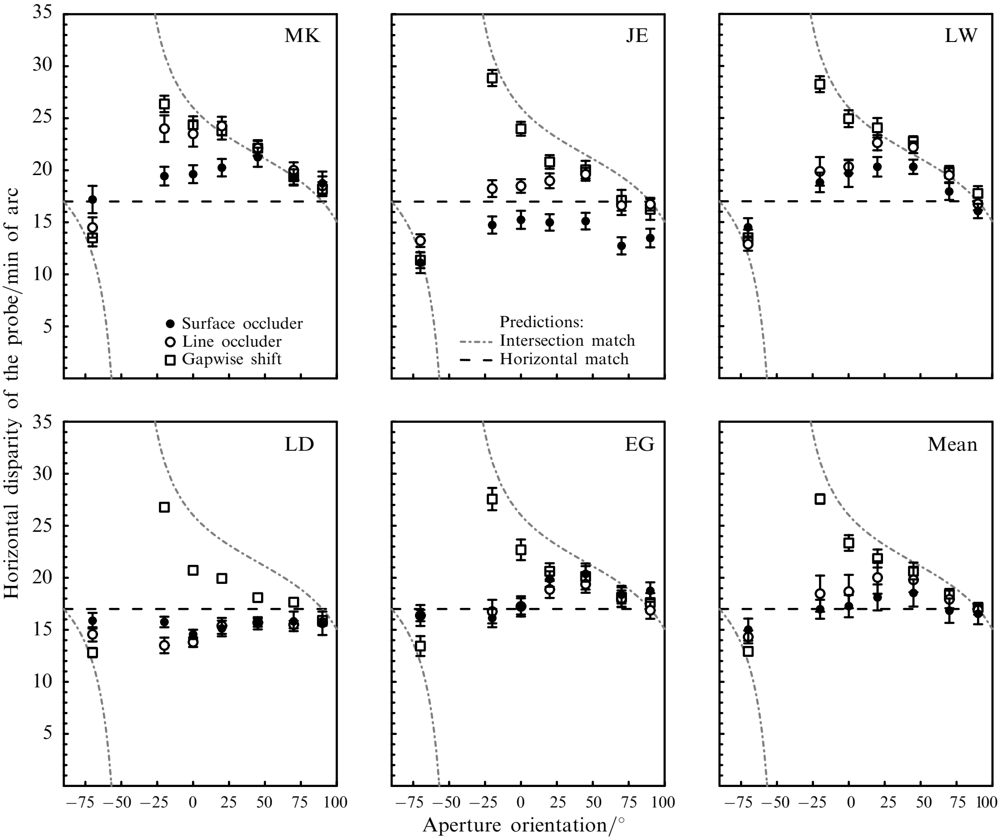

Figure 12. Results of experiment 2. The horizontal disparity of the probe for which the probe and the test line are perceived at the same depth plotted versus the aperture orientation (β in figure 11). Zero indicates a vertical aperture. Positive angles indicate that the aperture orientation is rotated anticlockwise from vertical. The black dashed lines show the prediction for the horizontal match and the grey dashed lines show the prediction for the intersection match. The singularity in the intersection-match prediction at an aperture orientation of $-45°$ arises because in this case the aperture and the test line have the same orientation and thus they never intersect. Error bars represent standard errors. For the surface occluder, matching occurred in a horizontal direction. For the line occluder, the results are subject-dependent (some subjects match horizontally and others according to the intersection match). The gapwise-shift results correspond quite well to the intersection-match prediction.

experiment 1.] This means that the visible endpoints of the test line were being matched, as predicted. These visible endpoints were located at the same positions in the stimulus where otherwise, had it been an occluder condition, the intersections between the test line and the occluder borders would have been. Thus, if indeed these intersection points in the stimuli for the surface-occluder condition were being matched, then the results for the surface-occluder condition and the gapwise-shift condition ought to be the same for all aperture orientations. However, the results for these two conditions are significantly different for a number of aperture orientations. This supports the conclusion that, for the surface-occluder condition, matching occurred in the horizontal direction.

The results for the line-occluder condition (open circles in figure 12) differ across subjects. For subjects LW, LD, and EG, these results are the same as the results for the surface occluder. They basically match the partially occluded test line in the horizontal direction (although for subject LW there seems to be an influence of the intersection points on the resulting match for aperture orientations of 20° and 45°). For subject JE, the results for the line-occluder condition appear to be shifted to a larger horizontal-probe disparity for all aperture orientations compared to the results for surface-occluder condition. The cause of this shift is not clear. Subject MK is inclined to match the intersection points. Although there are some significant differences between the results for the line-occluder condition and the intersection-match prediction, none of these results for the line-occluder condition is significantly different from the results for the gapwise-shift condition.

In sum, for the surface-occluder condition (in which the horizontal and vertical disparities of the occluders are well defined) matching of the test line occurs in the horizontal direction. For the line-occluder condition (in which matching of the occluders is ambiguous) the matching direction is subject-dependent and the intersections of the test line with the occluder borders play a role in binocular matching.

5 Control experiment: Aperture-imposed matching direction

We reasoned that a well-defined surface occluder on both sides of the aperture might have imposed a matching direction on our test line. Our surface occluders had clear features that the visual system could use for matching. The disparity of the occluders indicated a horizontal match (the half-images of the occluders were shifted horizontally on the screen). Also the occluders covered a relatively large area of the visual field (44 deg by 40 deg). If the horizontal-matching direction for the occluders would be imposed on our ambiguous test line, then it is not surprising that we found a horizontal match in our results. We conducted a control experiment in which the surface occluders were again presented on both sides of the aperture, but they no longer occluded the test line (either the test line was shorter or the width of the aperture was larger compared to the occluder conditions in the previous experiments). This allowed us to examine whether the test line was matched according to the shift of the visible endpoints of the test line, or according to a horizontal match imposed by the surface occluders on either side of the aperture.

5.1 *Stimuli*

In this control experiment the setup was the same as in experiment 1. There were again the surface occluders on both sides of a vertical aperture. The surface occluders always had a horizontal disparity on the screen of 26 min of arc. There were two aperture conditions. In the first one the test line had the same length as in experiments 1 and 2 (namely 3 deg). In this case, the aperture had a width of 5.8 deg. Note that this means that there were gaps between the surface occluders and the test line, and thus the test line was no longer occluded. In the other aperture condition, the width of the aperture was the same as in experiment 1 (2.1 deg), but the test-line length was decreased to 0.5 deg. In this second condition there were also no occlusions.

In each of these two aperture conditions there were four test-line shift conditions. In each condition the test line was shifted in a different way but the horizontal separation between the test-line half-images remained the same (17 min of arc). In one condition, the test-line half-images were shifted according to the gapwise shift, already described in experiments 1 and 2. In this condition, the endpoints of the test line were where the intersection points would have been had it been an occluding condition (thus the endpoints of the test line have a horizontal disparity component of 26 min of arc on the screen). In the second condition, the test-line half-images were shifted horizontally (the endpoints have a horizontal disparity of 17 min of arc). In the third, they were shifted perpendicularly to the orientation of the test line. In this case, the endpoints of the test line have a horizontal disparity component of 8.5 min of arc (half the half-images separation). In the last condition, the test-line half-images were shifted vertically, so the endpoints had a horizontal disparity component of 0 min of arc.

For every condition there were two staircase measurements, and the bias condition was also included, so in sum there were 18 interleaved staircases. In this control experiment only one subject participated (subject LD).

5.2 *Predictions*

What is the effect of the surface occluders with well-defined disparities on the matching direction for the test line? Van Ee and Schor (2000) showed, in a similar experiment in which the stimuli contained only the test line and the depth probe (thus without the occluders), that the assigned horizontal disparity of the test line corresponds to the horizontal disparity component of the endpoints of the test line, provided the test line is sufficiently short (see also the control conditions in experiments 1 and 2 in this paper).

One of the fundamental features of the conditions described above is that the endpoints of the test line are visible. If these local endpoints of the test line determine the match for the line, independently of the horizontal disparity of the surface occluders, then the probe settings would vary linearly with the horizontal disparity component of the endpoints.

If, however, the surface occluders impose a matching direction onto the test line, then in all conditions the test line should be matched horizontally and the probe disparity would always be set close to 17 min of arc (ie the horizontal separation between the test-line half-images).

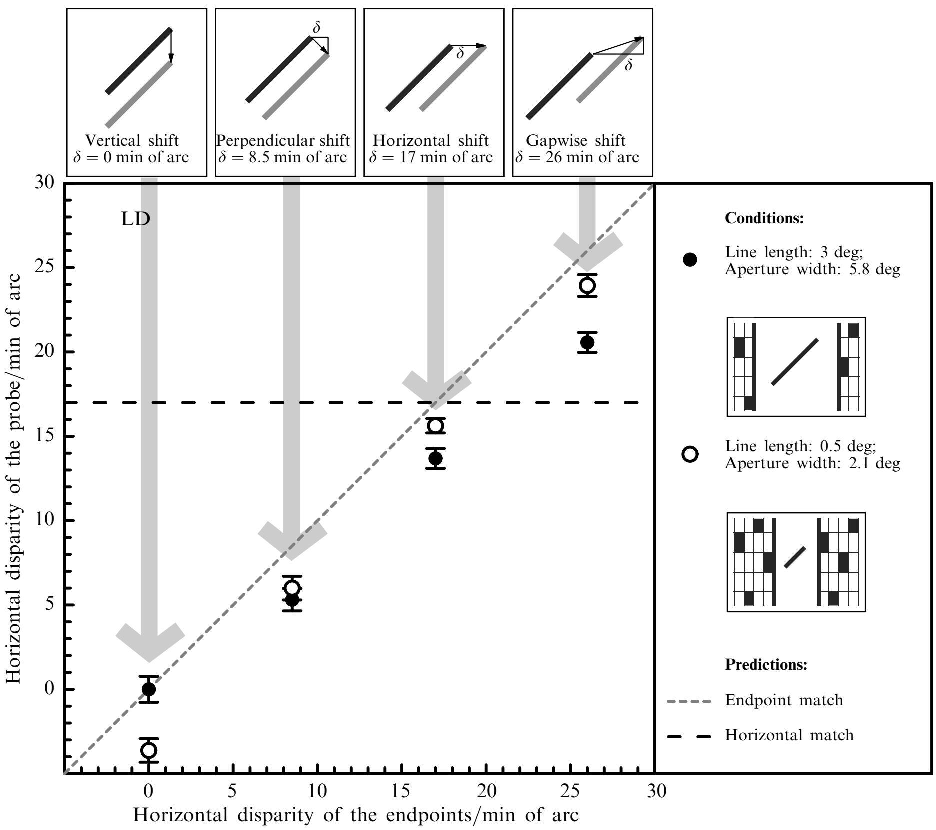

Figure 13. Results of the control experiment in which matching directions of test lines with visible endpoints were measured. Surface occluders with well-defined disparities were presented on both sides of the aperture. The horizontal disparity of the probe plotted against the horizontal disparity of the endpoints of the test line. (The test-line shifts are indicated in the boxes at the top.) Error bars represent standard errors. The results show that the test line was matched according to the shift of the endpoints.

5.3 *Results*

The results of the control experiment are presented in figure 13. The horizontal disparity of the probe is plotted against the horizontal disparity of the endpoints of the test line for each condition. The test-line shifts are indicated in the boxes at the top of the figure. The black dashed line shows the prediction for the horizontal match. The grey dashed line shows the prediction for the endpoint match. The filled circles represent the results for the condition in which the aperture width was 5.8 deg and the test-line length was 3 deg. The open circles represent the results for the condition in which the aperture width was 2.1 deg and the test-line length was 0.5 deg. The errors bars represent standard errors.

These results show that the local endpoints determine the matching direction for the test line. All the results are close to the prediction for the endpoint match, implying that the horizontal disparities of our surface occluders do not necessarily impose a horizontal match on the test line. We conclude that the results we found in experiments 1 and 2 are not simply imposed by the horizontal-matching features of the surface occluders and that local matching of the test line can have matching directions that differ from the more global matching of these surface occluders.

6 Discussion

We have quantitatively examined binocular matching for a partially occluded test line. We have done so using a depth-probe method (Mitchison and McKee 1987; van Ee and Schor 2000), which means that we have analysed binocular matching at a geometrical level (as opposed to a mechanistic neural level). We summarised four possible matching hypotheses. The first one implied that binocular matching of partially occluded objects occurs in the horizontal direction. Our results show that, indeed, the matching direction is horizontal when the horizontal and vertical disparities of the occluders are well defined. The horizontal match is supported by the work of Anderson (Anderson 1994, 1999b; Anderson and Nakayama 1994; Anderson and Sinha 1997). He investigated the perception of subjective occluding contours (subjective meaning that the occluder was not explicitly defined in the stimulus, eg by contrast). He explained his findings by assuming that matching occurs in the horizontal direction. He reported that, in order to perceive the subjective occluding contours, reference points on the occluded object as well as in the surround are needed. When these reference points are absent, the subjective occluding contours are no longer perceived. This dependence on the presence of reference points is rather similar to the dependence on well-defined disparities of the occluders in our study. When the disparities of the occluders are ill-defined, the partially occluded line is not necessarily matched in the horizontal direction. In this case, the intersections of the test line and the occluder borders play a role as shown by our experiment 2.

Consider again the oblique line behind an aperture (figure 1). Farell (1998) suggested that the half-images of the line (figure 1b) may be matched in a variety of directions. He stated: "in naturalistic, 'layered' scenes of occluding and transparent surfaces, stereo correspondence becomes a two-dimensional (2-D) matching problem in which horizontal disparity is not a reliable cue to depth". In his stereoplaids the intersections of its component gratings, rather than the horizontal disparities of the gratings, could be responsible for the perceived depth. For, if the visual system would prefer to compute the perceived depth from the horizontal shifts of the 1-D components (the gratings) only, it would have decomposed the stereoplaids into two separate gratings with different horizontal disparities, rather than fusing the gratings to form one stereoplaid in one depth plane. For our aperture problem such a mechanism would suggest that the perceived depth of the line is related to the shift of the intersections of the line with the occluder (so according to our intersection-match hypothesis). Our results show that, indeed, the intersection points can determine the matching direction for the test line, but only when the matching direction for the occluders is ambiguous. This is consistent with the reported observations of Farell (see figure 4 for a demonstration of his effects), since for his stereoplaids, too, matching was ambiguous for both gratings (ie for the occluding as well as for the occluded lines). In addition, it is worth noting that for Farell's stimulus (figure 4c) we found considerable variance across observers, just as for our ambiguous stimuli, and with prolonged viewing some observers were able to perceive both a depth percept based upon intersection matches and a percept based upon horizontal matches.

According to the aperture-direction match, the test-line half-images would be matched in the direction of the main axis of the aperture. Note the similarities between the aperture problem for motion (barber-pole effect) and the aperture problem for stereopsis (Shimojo et al 1988; Anderson 1994; Anderson and Sinha 1997). These similarities would suggest that matching two images at separate moments in time is rather similar to matching the images of the two separate eyes. However, our results indicate that binocular matching does not occur along the main axis of the aperture. In addition, it has been reported that the barber-pole effect for motion is not present when the barber-pole lines have uncrossed disparity relative to the occluders (Shimojo et al 1989; Anderson 1999b). In this case, the lines appear to be moving in a direction perpendicular to their own orientation. This, together with our results, indicates that the analogy between matching for motion detection and matching for stereopsis is limited.

7 Conclusion

So far, no studies exist in the literature that quantitatively reveal the disparity that is assigned to a partially occluded line in the aperture problem. Our results show that, from a geometrical point of view, matching of partially occluded objects is horizontal when the occluders are well-defined in terms of horizontal and vertical disparity (in agreement with Anderson 1994). However, if the visual input lacks this information, then the matching direction for the occluders becomes ambiguous and the intersections of the test line and the occluder borders play a role in the determination of the perceived depth of the test line (in agreement with Farell 1998). In the latter case, the matching direction of the test line varies with aperture orientation and there is a relatively large variability across subjects.

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