The Auditory System and Human Sound-Localization Behavior
Answers Exercises Chapter 1: Introduction.

Exercise 1.1:

In Figure 1.7A, the iso-elevation and iso-azimuth lines are indicated as parallel small circles on the unit sphere. For a target in the frontal hemifield at fixed azimuth α_0 deg, the iso-azimuth circle on the sphere has radius $\cos(\alpha_0)$. For straight ahead, $\alpha_0 = 0$, the radius is 1.0, and the target's elevation angles can run over the full range from $[-\pi/2, +\pi/2]$. At the far-lateral positions, $\alpha_0 = \pm\pi/2$, the radius is zero, and hence all elevation angles are confined to zero deg. At the intermediate azimuths, the allowed elevations will run from $[-\pi/2+\alpha_0, +\pi/2-\alpha_0]$. Indeed, this behavior is described by

$$\sin\varepsilon \in [-\cos\alpha_0, +\cos\alpha_0]$$

The same analysis can be applied to targets at a fixed elevation, ε_0: now the azimuth angles can run from $[-\pi/2, +\pi/2]$ for the straight-ahead direction, and they are constrained to zero at the zenith and nadir. For the intermediate elevations, the azimuths can run between

$$\sin\alpha \in [-\cos\varepsilon_0, +\cos\varepsilon_0]$$

Note that always (see also Chapter 1)

$$|\alpha + \varepsilon| < \pi/2$$

Exercise 1.2:

Eqn. (1.2) reads for the coordinates of the auditory and visual stimulus in a spatial reference frame:

$$A_S = A_H + a \cdot E_H + b \cdot H_S \quad \text{and} \quad V_S = V_E + c \cdot E_H + d \cdot H_S$$

For an adequate transformation the coefficients should be:

$$[a, b, c, d] = [0, 1, 1, 1]$$

Thus, the auditory input does not require information about the eye position, whereas the visual input does. Now, if the spatial coordinates of the two targets are identical (and the temporal coincedences of their occurrence are within a required window) the two stimuli likely arose from a single space-time object and therefore should be integrated. This is the case when

$$A_H + H_S = V_E + E_H + H_S \Leftrightarrow A_H = V_E + E_H$$

with A_H the auditory sensory coordinates (re. head), V_E the visual coordinates on the retina, and E_H the eye orientation in the head.

Now suppose that the sensory coordinates are the same: $A_H = V_E$. In that case, the stimuli can only emerge from the *same* object in external space (and time) when the eye position is zero! As soon as the eye is off-center, audio-visual integration should not occur..... However, when the eye position is eccentric, integration should occur whenever the above equation is met (so: a difference in sensory coordinates by exactly the eye-position signal).

Exercise 1.3:
The derivative of the perturbed function:

$$f_n(x) = f(x) + \frac{\sin(nx)}{\sqrt{n}}$$

is given by

$$\frac{df_n(x)}{dx} = \frac{df}{dx} + \sqrt{n} \cdot \cos(nx)$$

Clearly, for n arbitrarily high, the derivative increases without bound. Hence, calculating the derivative of $f(x)$ is an ill-posed problem, even though the perturbation goes to zero for $n \to \infty$!

Exercise 1.4:
Only the output neurons with a net excitatory input may produce activity. This concerns neurons 3, 5, 7 and 9. All other neurons (nrs. 1, 2, 4, 6, 8, 10 and 11) only receive inhibition from the total input pattern, which is given by: $-0.1 \cdot (1 + 3 + 2 + 1) = -0.7$.
The net input for the four excited neurons is:

$y_3 = 1-0.3-0.2-0.1 = +0.4$ $y_5 = 3-0.1-0.2-0.1 = +2.6$ $y_7 = 2-0.3-0.1-0.1 = +1.5$ $y_9 = y_3$

So, if the excitatory threshold for the output neurons is > 1.5, only the neuron with the strongest input, here y_5, will fire.