The Auditory System and Human Sound-Localization Behavior

Exercises Chapter 3

Problem 3-1: linear!

Verify the following important corollary of the superposition principle, Eqn. 3.8: suppose a linear system is stimulated with step inputs of different amplitudes, i.e.:

$$x(t) = A \cdot U(t) \text{ with } A \in \mathbb{R}$$

and that the unit-step response of the system (to A=1) is given by *s(t)*. After a finite time (defined as the response duration, *D*) the step response becomes (and remains) constant (i.e., $ds/dt = 0$, $\forall t>D$). Then for any linear system:

(i) *D* is independent of input amplitude *A*.

(ii) The peak velocity of the step response increases linearly with *A*.

Problem 3-2: linear?

A ball, dropped at t=0 from height h_0 to the earth, obeys the following linear differential equation:

$$F = m \cdot g = m \cdot \frac{d^2y}{dt^2}$$

Verify that the solution for this problem is:

$$y(t) = h_0 + v_0 \cdot t - \tfrac{1}{2}g \cdot t^2$$

The duration of the fall towards the earth's surface is then determined by (suppose that the initial velocity $v_0=0$):

$$D = \sqrt{\frac{2h_0}{g}}$$

So, although the system is described by a linear differential equation, the response duration depends on the input amplitude, initial height h_0. Thus, in line with what we just have seen in Problem 3-1, we have to consider the system to be *nonlinear*! Do you agree with this statement? Why (not)?

Problem 3-3: HP filter

Analogous to the analysis of the low-pass filter, described in section 3.4, we here consider the first-order High-Pass (HP) filter. This system can simply be modeled by taking the voltage across the resistor as output of the RC-circuit of Fig. 3.10A.

(a) Show that its impulse response is given by:

$$h_{HP}(\tau) = \delta(\tau) - \frac{1}{T_{HP}} \exp\left(-\frac{\tau}{T_{HP}}\right) \quad \text{with } T_{HP} = RC, \text{and } \tau \geq 0$$

Give also the HP step response.

(b) Show that its Gain and Phase characteristics are given by:

$$G_{HP}(\omega) = \frac{\omega \cdot T_{HP}}{\sqrt{1 + \omega^2 T_{HP}^2}} \quad \text{and } \Phi(\omega) = \arctan\left(\frac{1}{\omega T_{HP}}\right)$$

(c) Also note from Fig. 3.10A that HighPass = AllPass – LowPass (from Kirchhoff's law) which provides a shortcut to obtain the results of (a) and (b).

(d) The slow-phase eye-movement response of the Vestibular Ocular Reflex (VOR) is well approximated by such a filter. The time constant of the VOR is about T_V~20 s. Plot the characteristics (as a Bode plot) and verify its High-Pass properties.

Problem 3-4: Integrator

Without having to use advanced mathematics and Fourier analysis it is sometimes straightforward to compute the transfer characteristic of linear systems. Try the following example, which is the pure integrator:

$$y(t) = \int_0^t x(\tau) d\tau$$

(a) Show that this is indeed a linear system.

(b) What is its impulse response?

(c) What is the step response?

(d) Determine amplitude and phase characteristic of this system (no need for FT!)

(e) Compare to the first-order LP system. Why is the LP system of Fig. 310A also called a *'leaky integrator'*?

Problem 3-5: Differentiator

Consider the differentiator:

$$y(t) = \frac{dx}{dt}$$

Answer the same questions (a) – (d) as in Problem 3-4.

(e) Compare the differentiator with the HP system of Problem 3-3.

Problem 3-6: Band pass and band stop

(a) Investigate how to create either a Bandpass (BP) or a Bandstop (BS) second order filter by combining a first-order LP filter (time constant T_{LP}) with a first-order HP filter (time constant T_{HP}). How would you create an Allpass filter with the LP and HP filters?

(b) Determine the gain and phase characteristics for the BP, BS and AP filters.

Problem 3-7: Series concatenation

(a) Calculate the impulse response function by using convolution for the series concatenation of two first-order LP filters, described by:

$$h_1(\tau) = \frac{\exp\,(-\frac{\tau}{T_1})}{T_1} \quad \text{and} \quad h_2(\tau) = \frac{\exp\,(-\frac{\tau}{T_2})}{T_2} \quad \text{with} \;\; T_1 > T_1$$

Plot this impulse response function.

(b) Determine (and plot) the transfer characteristic of this system.

Problem 3-8: Series

Calculate the impulse response, step response, amplitude- and phase characteristic of the series concatenation of a pure integrator, followed by a scalar gain, *G*, and a pure time delay of *T* s.

Problem 3-9: Feedback

Consider the circuit in Fig. 3.18. Each of the four subsystems is linear, and their transfer characteristics are given by $H_1(\omega)$, $H_2(\omega)$, $H_3(\omega)$, and $H_4(\omega)$, respectively. Suppose that $H_2(\omega)$ is a pure integrator.

(a) Determine the transfer characteristic of the total system. Your answer may only contain the characteristics of the three subsystems. Express also the amplitude and phase characteristics of the total transfer as function of the amplitude and phase characteristics of the three subsystems.

(b) When will the system become unstable?

(c) Suppose that $H_3(\omega)$ is a constant gain, G. Which requirements should hold for system $H_4(\omega)$ to keep the total system unstable?

(d) Suppose that also $H_4(\omega)$ is a constant gain, A. For which frequency will the system be unstable?

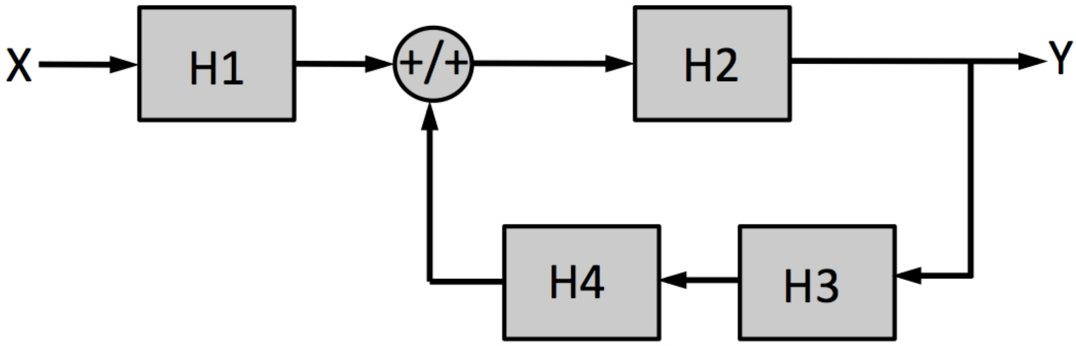

Figure 3.18 *A positive feedback scheme consisting of four linear subsystems.*

Problem 3-10: Difference differentiation algorithm

In a computer, you would approximate the ideal differentiator of Problem 3-5 by the difference algorithm:

$$\dot{y}\,(t) \approx \frac{y(t+\Delta T) - y(t)}{\Delta T}$$

(a) By doing so, however, you filter the original signal. Show this by computing the transfer characteristic of this operation.

(b) To reduce noise in an analog signal we add a LP filter (time constant T) in series with a pure differentiator (from Problem 3-5). Determine the transfer characteristic of the total system; give the Bode plot, and analyze how the results depend on T.

Problem 3-11: Feedback

Here we analyze the influence of a delay on the transfer characteristic of a linear system with feedback (Fig. 3.19).

(a) Determine the Laplace transform of a pure delay: $y(t) = x(t-\Delta T)$, and from that the transfer characteristic (in the frequency domain) of the delay.

(b) Take the system in Fig. 3.19. Determine the total transfer function and the loop gain. The system will spontaneously oscillate, and become unstable, when the loop-gain exceeds the value of 1, and at the same time has a phase shift of -180°. Make a Bode analysis of the system and estimate the frequency ω_0 where instability kicks in.

(c) What happens to the system if A is increased/lowered? What if the time constant T is increased/lowered?

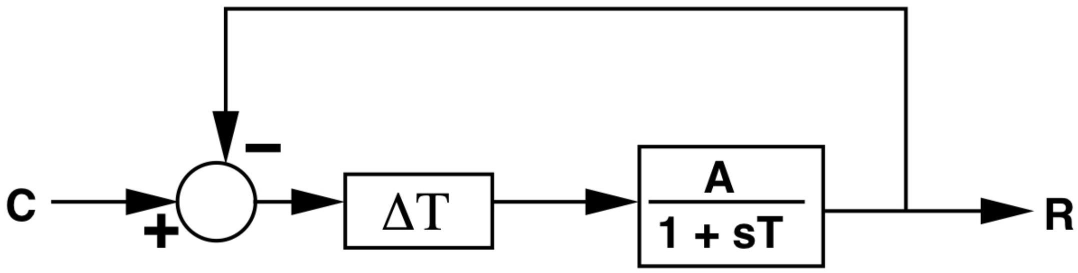

Figure 3.19 *Negative feedback system of a first-order low-pass filter with a delay.*

***Problem 3.12: Echoes**

Consider the situation of the free field with a sound source and a microphone. Without reflectors in the environment the microphone will record the sound pressure signal *p(t)*. Suppose the presence of a reflector, which produces an echo of the sound source. Assume that the echo is a filtered (impulse response *a(τ)*) and delayed (delay ΔT) version of the original sound, *p(t)*.

(a) Write a time-domain expression for the signal *p*(t)* that is picked up by the microphone.

(b) Determine the transfer characteristic that relates the original input sound spectrum, *P(ω),* to the measured spectrum with echo *P*(ω)*:

$$T(\omega) \equiv P^*(\omega)/P(\omega)$$

(e) The same as in (a) and (b) for the situation of N different reflectors, each with their own impulse response, $a_k(\tau)$ and delay, ΔT_k.

(f) For $N = 1$, how can the distance to the reflector be determined from the power spectrum of the transfer characteristic, $|T(\omega)|^2$?

(g) Draw the power spectrum of white noise with an echo having a 1 ms delay.

Problem 3.13: Autocorrelation

Calculate the auto-correlation functions for the following examples:

(a) $x(t) = P$ for $-T \leq t \leq +T$ and $x(t) = 0$ elsewhere

(b) $x(t) = A \cdot \cos \omega t$

(c) $x(t) = A \cdot \exp\left(-\frac{t}{T}\right)$ for $t \geq 0$ and $x(t) = 0$ elsewhere

(d) $x(t) = \begin{cases} A \cdot \left(\text{sgn}(t) \cdot 1 - \frac{t}{T}\right) & \text{for } -T \leq t \leq +T \\ 0 & \text{elsewhere} \end{cases}$

***Problem 3.14: Auto- and cross-correlation functions**

For the simple LP filter of Fig. 3.20 we have seen that the relation between the input (*x(t)*=GWN) and output, *y(t)*, is given by:

$$y(t) = \int_0^\infty \frac{1}{RC} \exp\left(-\frac{\tau}{RC}\right) \cdot x(t-\tau) d\tau$$

(a) Determine the cross-correlation function $\phi_{yx}(\sigma)$

(b) Determine the auto-correlation function of the output $\phi_{yy}(\sigma)$

(c) Determine the response, *y(t)*, if *x(t)* = the pulse from Problem 3-13a

(d) Determine the cross-correlation function $\phi_{yx}(\sigma)$ for the pulse input.

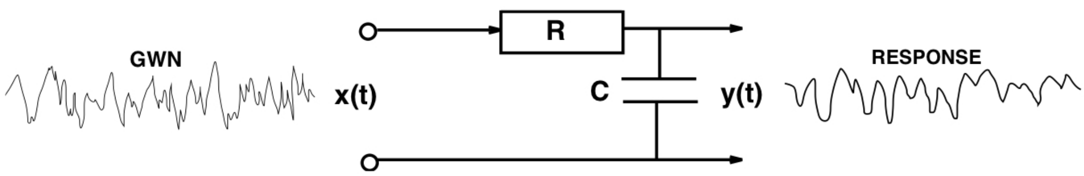

***Figure 3.20** First-order low-pass filter with GWN as input.*