The Auditory System and Human Sound-Localization Behavior

Exercises Chapter 9

Problem 9-1 From Eqn. 9.1 it is possible to estimate the total number of cones in the human retina. Assume that the eye is a sphere with a radius of 1.2 cm, and that the retina covers the entire backside of the spherical surface. Assume that cells are distributed in a rotation-symmetric way around the fovea. Write down and solve the (2D) integral equation for the total number of retinal cones (*note: number of cells = cell density × surface; use polar coordinates, (r,φ)*).

Problem 9-2

a Approximate a saccade velocity profile by a simple triangle (base = duration, height = peak velocity, at fixed T_{pk}). Comment on Eqn. 9.4d.

b Also try to derive Eqn. 9.4d from Eqns. 9.4a,b.

c When the duration increases according to Eqn. 9.4a, predict $V_{pk}(R)$ from the simple triangular approximation.

Problem 9-3 Refer to the superposition principle described in Chapter 3, to argue why Eqns. 9.6 have to apply for *any* linear system.

Problem 9-4

a Apply the LT on Eqn. 9.7 to demonstrate Eqn. 9.8, where

$$C = \frac{k_1 + k_2}{k_1 k_2}, \quad T_z = \frac{r_1 + r_2}{k_1 + k_2}, \quad T_1 = \frac{r_1}{k_1}, \quad T_2 = \frac{r_2}{k_2}$$

b By applying inverse LT, show that the temporal behavior of the impulse response in the time domain is independent of T_z, and given by:

$$h(\tau) = \frac{1}{T_1 - T_2}\left[(1 - T_z/T_1)\exp\left(-\frac{\tau}{T_1}\right) + (T_z/T_2 - 1)\exp\left(-\frac{\tau}{T_2}\right)\right]$$

Problem 9-5 Apply the inverse reconstruction technique to the Goldstein-Robinson model to derive Eqn. 9.14.

Problem 9-6 Here we analyze the cross-inhibitory neural network of Fig. 9.14, and show that it performs a perfect neural integration on the *difference* signal between the left and right inputs, thereby getting rid of the d.c. components in the input. Suppose that the neurons have the following input-output relationships (in Laplace notation):

$$X_{R,L}(s) = \frac{IN_{R,L}(s)}{s \cdot \tau + 1} \quad \text{with} \quad IN_{R,L}(s) = U_{R,L}(s) - W \cdot X_{L,R}(s) \text{ and } \tau = 5\text{ms}$$

$U_{L,R}(s)$ is the push-pull input to the neurons (when U_L goes up, U_R goes down, and v.v.), and is described by

$$U_{L,R} = \bar{U} \pm \Delta U$$

Now show (use an appropriate change of variables to uncouple the pair of equations) that as W→1, the output of the neurons can be written in the following way:

$$X_{L,R}(s) = \frac{\bar{U}/2}{s \cdot \frac{\tau}{2} + 1} \pm \frac{\Delta U}{s}$$

In other words, the dc disappears at the output with a time constant of 2.5 ms, but the change in firing is integrated perfectly (T→∞)!

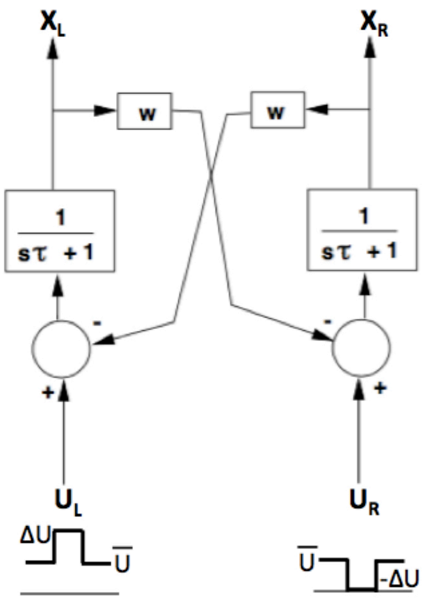

Figure 9.14 *Cross-inhibitory neural network to model the neural integrator.*

Problem 9-7 Repeat the analysis of Eqn. 9.17 for an arbitrary gain in the direct pathway of the PSG. Determine the brainstem transfer characteristic, and analyze (in the time domain) what happens to the eye movements when $T' = T \pm \Delta T$.

Problem 9-8 Include the slide pathway in the brainstem Pulse-Slide-Step Generator, and determine the transfer characteristics of each of the three paths, such that the total model from pulse generator to eye movement implements a pure integrator.

Problem 9-9 Here we will solve the Scudder model of Fig. 9.6 analytically. The SC burst is described by a rectangular pulse: *SC(t)=P* for $0 \leq t \leq D$ and 0 elsewhere, and P▪D = ΔE (the desired saccade amplitude). The MLB nonlinearity is $v(t) = v_{max} \cdot (1 - \exp(-\beta m(t)))$, with m(t) the dynamic motor error. Assume that the omnipause trigger is given at t=D.

a Show that the dynamic motor error obeys the following differential equation:

$$\frac{dm}{dt} = \Delta E - v_{max} \cdot (1 - \exp(-\beta m))$$

b Solve this equation for m(t).

(Hint: use substitution: s = exp(*-βm)-1* and first solve for s).

c Use the definition of dynamic motor error to determine the instantaneous eye displacement of this model, Δe(t), and show that for t→∞ the eye reaches the desired displacement amplitude.

Problem 9-10 In the 'Common Source' model, the brainstem is driven by a vectorial pulse generator that transforms the amplitude of a 2D motor error vector, Δe_{vec}, into a vectorial eye-velocity command according to:

$$\dot{e}_{vec}(t) = v_{max} \cdot \left[1 - \exp\left(-\frac{|\Delta\vec{e}_{vec}|}{m_0}\right)\right]$$

with |x| the magnitude of the vector. Give expressions for the velocity profiles of identical horizontal saccade components for the situations sketched in Fig. 9.7C (top panel; i.e. the component amplitude is fixed, but the vector rotates over angle φ with respect to the horizontal direction).