Modelling 3D saccade generation by feedforward optimal control

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Outline

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Motivation

- Saccades (rapid eye movements)
- Extra degrees of freedom
- Existing neurophysiological models seldom work with 3D saccades and rarely consider both kinematic and dynamic properties.
- Trying to explain the kinematics and nonlinear dynamics using open loop optimal control of a simplified 3D biomimetic model.
Extraocular Muscles

- 6 extraocular muscles (3 agonist-antagonist pairs).
- 5 of the muscles start at the annulus of Zinn. The inferior oblique muscle starts near the nose.
- The muscle actions are coupled.
Sectional View of an Eye Saccade

MRI scan of right eye for Saccade from -30 deg to +40 deg
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4981490/
Donders’ and Listing’s Law

- Donders’ Law: The eye's 3D orientation is uniquely determined by its 2D gaze direction and this orientation does not depend on the history of eye motions.

- When eye orientations are represented as single axis \( r = [r_x, r_y, r_z] \) rotations from a primary gaze direction it was observed that this axis lies in a plane of ~0.6 deg thickness.
Nonlinear Dynamics

- Skewed velocity profiles
- Saturating peak velocities
- Main sequence

A. Human velocity profiles

B. S<0.9 Fast saccades
   0.9<S<1.1
   S>1.1 Slow saccades

C. k=1.70
   Slow and fast saccades
Model

- Only included their most essential features
- Mechanical model with 3 motors.
- Muscle pairs- elastic cables attached to each motor.
- Routing points for cables (pulleys).
Feedforward Optimal Control

- Linearized the system using Matlab system identification
- Feedforward optimal control with the motor rotations U as system input.

\[
\begin{align*}
\text{minimize } & \quad J_{TOT} = \sum_2 \lambda_n J_n(U, D, R) \\
\text{subject to } & \quad x_{t+1} = Ax_t + Bu_t \\
& \quad r_t = Cx_t + Eu_t \\
& \quad t = 0, ..., D
\end{align*}
\]

- The costs considered were a combination of:
  - Accuracy cost \( J_A(r) = \left( r_y, D - \hat{r}_y \right)^2 + \left( r_z, D - \hat{r}_z \right)^2 \) & \( \hat{r} = 0 \)
  - Energy cost \( J_E(U) = \| \Delta U \|^2 \)
  - Static force cost \( J_F(r) = r_p^T H F r_p \)
  - Duration cost, \( J_D(p) = \left( 1 - \frac{1}{1 + \beta p} \right) \)
  - Listing plane cost \( J_L(r_x) = \| r_x \|^2 \)
Feedforward Optimal Control
Results

- Dropping either the penalty on saccade duration ($J_D$) or energy ($J_E$) disrupts the main-sequence and cross-coupling properties of the saccades.

- Because of the strong component cross-coupling, saccade trajectories were approximately straight in all directions.

- By including the minimization of static total force on the eye, Listing’s law emerged without any additional assumptions regarding the eye’s cyclo-torsional state.
Thank You.
Questions or Suggestions?