

# Modelling 3D saccade generation by feedforward optimal control

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**LARSyS**

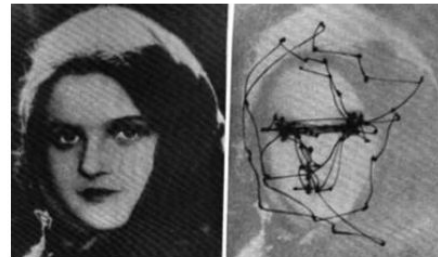
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# Outline

1. Motivation
2. Human Eye
3. Model
4. Optimal Control
5. Results

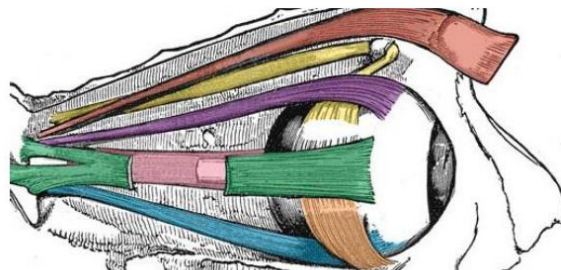
# Motivation



- Saccades ( rapid eye movements)
- Extra degrees of freedom
- Existing neurophysiological models seldom work with 3D saccades and rarely consider both kinematic and dynamic properties.
- Trying to explain the kinematics and nonlinear dynamics using open loop optimal control of a simplified 3D biomimetic model.

# Extraocular Muscles

- 6 extraocular muscles (3 agonist-antagonist pairs).
- 5 of the muscles start at the annulus of Zinn. The inferior oblique muscle starts near the nose.
- The muscle actions are coupled.

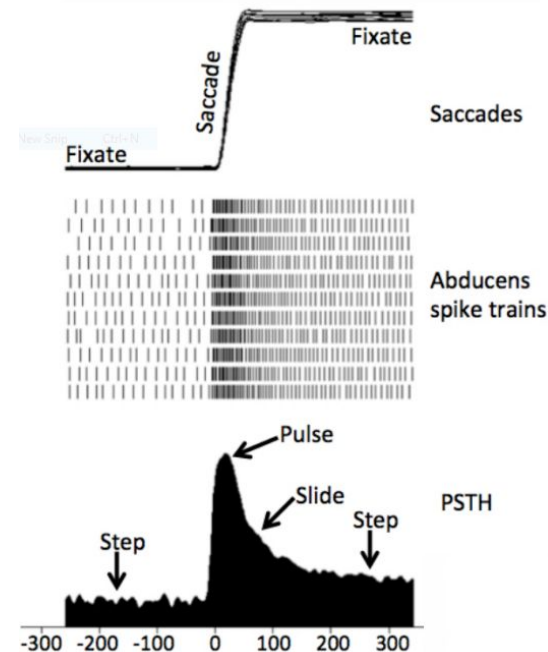


- Superior oblique
- Inferior oblique
- Superior rectus
- Medial rectus
- Lateral rectus
- Inferior rectus

# Sectional View of an Eye Saccade

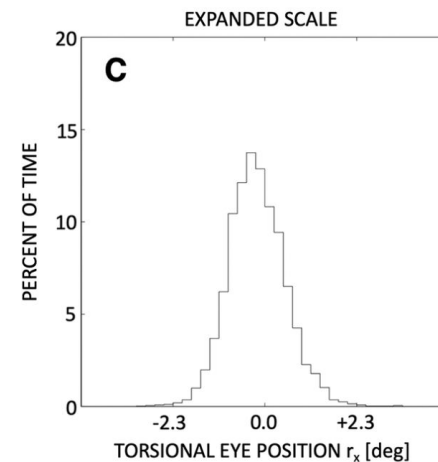
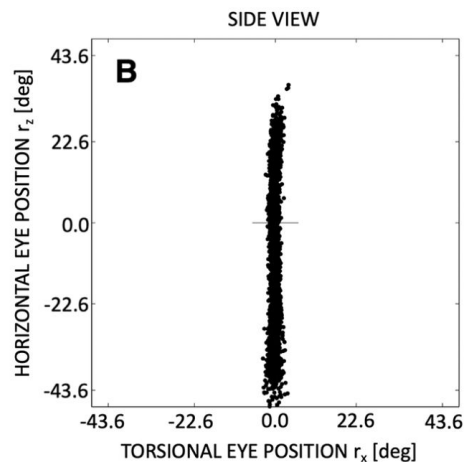
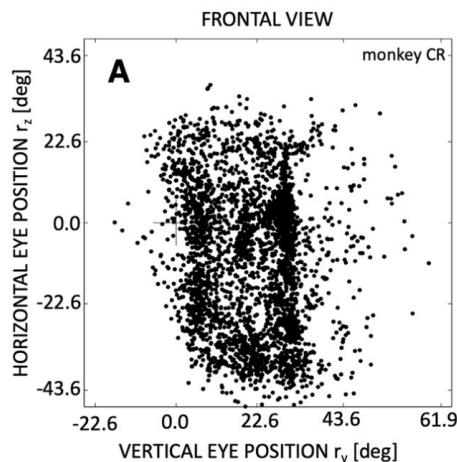


MRI scan of right eye for Saccade from -30 deg to +40 deg  
<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4981490/>



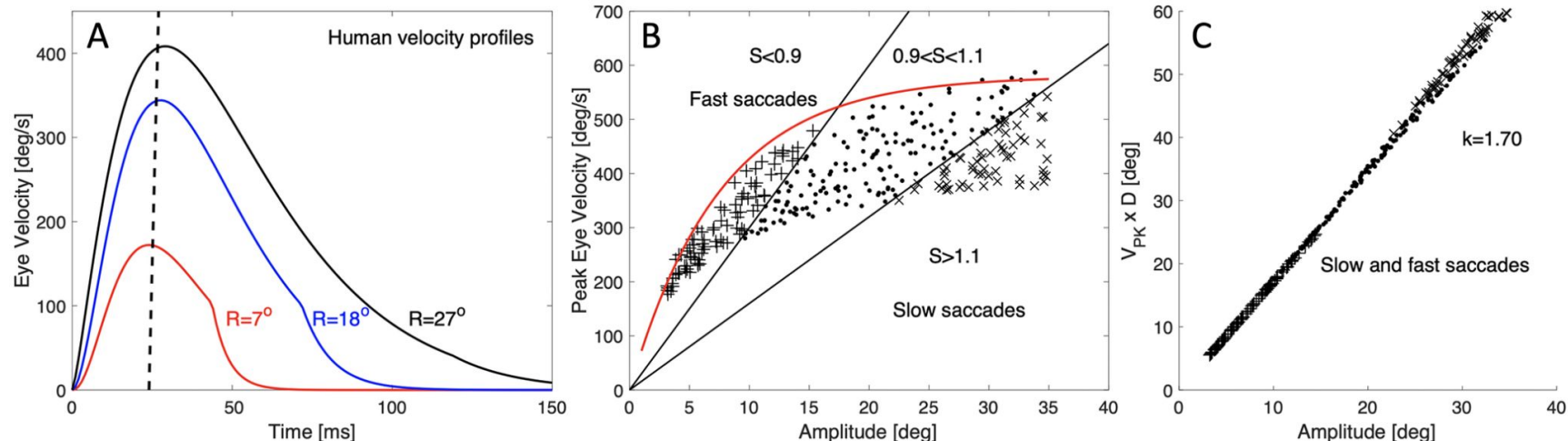
# Donders' and Listing's Law

- Donders' Law: The eye's 3D orientation is uniquely determined by its 2D gaze direction and this orientation does not depend on the history of eye motions.
- When eye orientations are represented as single axis ( $r = [r_x, r_y, r_z]$ ) rotations from a primary gaze direction it was observed that this axis lies in a plane of  $\sim 0.6$  deg thickness.



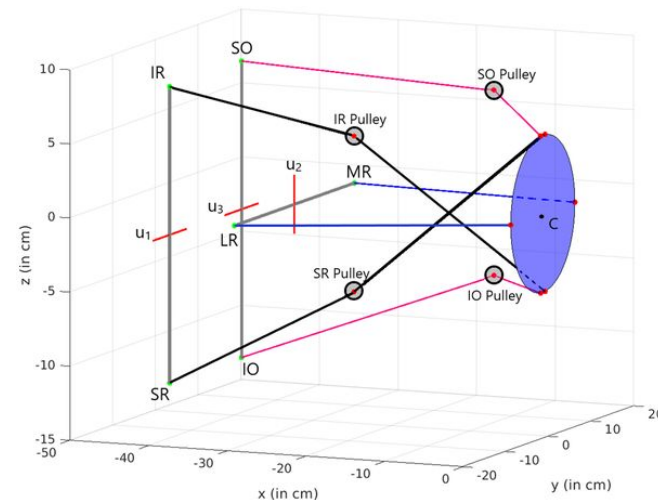
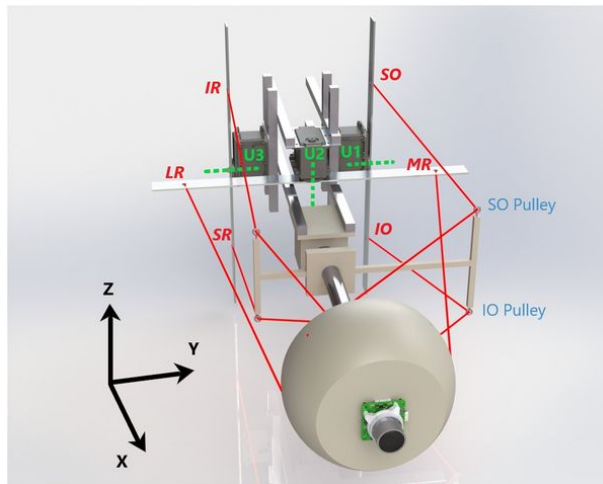
# Nonlinear Dynamics

- Skewed velocity profiles
- Saturating peak velocities
- Main sequence



# Model

- Only included their most essential features
- Mechanical model with 3 motors.
- Muscle pairs- elastic cables attached to each motor.
- Routing points for cables(pulleys).





# Feedforward Optimal Control

- Linearized the system using Matlab system identification
- Feedforward optimal control with the motor rotations  $U$  as system input.

$$\text{minimize}_{\mathbf{U}, D} \quad J_{TOT} = \sum_x \lambda_x J_x(\mathbf{U}, D, \mathbf{R})$$

$$\text{subject to :} \quad \mathbf{x}_{t+1} = A\mathbf{x}_t + B\mathbf{u}_t$$

$$\mathbf{r}_t = C\mathbf{x}_t + E\mathbf{u}_t \quad t = 0, \dots, D$$

- The costs considered were a combination of:

- Accuracy cost  $J_A(r) =$

$$\left( r_{y,D} - \hat{r}_y \right)^2 + \left( r_{z,D} - \hat{r}_z \right)^2 \quad \& \quad [\dot{r}, \ddot{r}] = 0$$

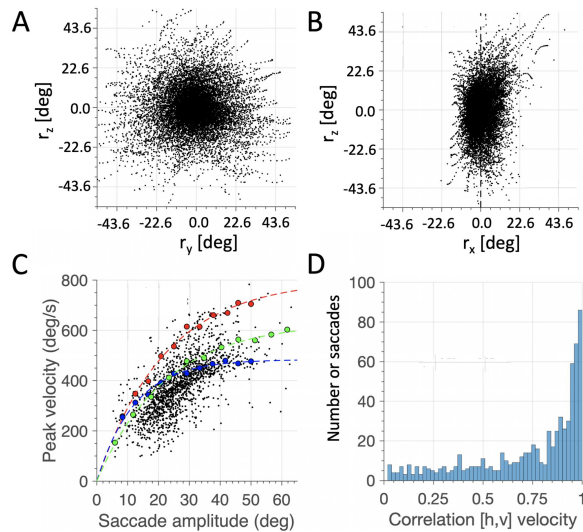
- Energy cost  $J_E(U) = \left\| \Delta U \right\|^2$

- Static force cost  $J_F(r) = r_p^T H_F r_p$

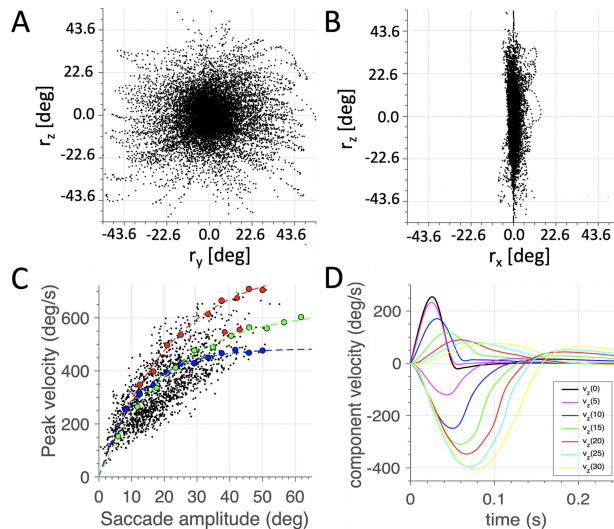
- Duration cost,  $J_D(p) = \left( 1 - \frac{1}{1 + \beta p} \right)$

- Listing plane cost  $J_L(r_x) = \left\| r_x \right\|^2$

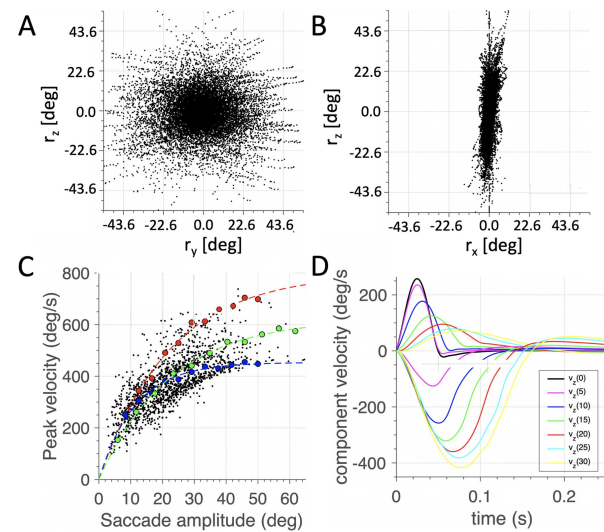
# Feedforward Optimal Control



J<sub>AED</sub>



J<sub>AEDL</sub>



J<sub>AEDF</sub>

# Results

- Dropping either the penalty on saccade duration ( $J_D$ ) or energy ( $J_E$ ) disrupts the main-sequence and cross-coupling properties of the saccades.
- Because of the strong component cross-coupling, saccade trajectories were approximately straight in all directions.
- By including the minimization of static total force on the eye, Listing's law emerged without any additional assumptions regarding the eye's cyclo-torsional state.

Thank You.  
Questions or Suggestions?