

## **Optimization of cortical hierarchies with continuous scales and ranges**

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## **Abstract**

Although information flow in the neocortex has an apparent hierarchical organization, there is much ambiguity with respect to the definition of such a hierarchy, particularly in higher cortical regions. This ambiguity has been addressed by utilizing observable anatomical criteria, based upon tract tracing experiments, to constrain the definition of hierarchy (Felleman and Van Essen, 1991). There are, however, a high number of equally optimal hierarchies that fit these constraints (Hilgetag *et al.*, 1996). Here, we propose a refined constraint set for optimization which utilizes continuous, rather than discrete, hierarchical levels, and permits a range of acceptable values rather than attempting to fit fixed hierarchical distances. Using linear programming to obtain hierarchies across a number of range sizes, we find a clear hierarchical pattern for both the original and refined versions of the Felleman and Van Essen (1991) visual network. We also obtain an optimal hierarchy from a refined set of anatomical criteria which allows for the direct specification of hierarchical distance from the laminar distribution of labelled cells (Barone *et al.*, 2000), and discuss the limitations and further possible refinements of such an approach.

## **Introduction**

A primary aim of cognitive neuroscience is to elucidate the means by which the structural and functional organization of the brain gives rise to its capacity for high-level information processing. Towards this end, various mammalian nervous systems have been subjected to a progression of anatomical parcellation schemes ranging from the gross scale (spinal cord, hindbrain, midbrain, cortex) to the fine (nuclei, ganglia, cortical regions, hypercolumns, microcolumns). In each of these schemes there is an inherent hierarchical organization of information flow which appears to progress from discrete bits of sensory information, to increasingly refined integrative representations of these

bits, to (in higher mammals) a semiotic classification of environmental entities and the probabilistic relationships between them. The literature underlying this abstract description of the brain's hierarchical organization of information processing is expansive; however, a few individual lines of research are especially prominent. An obvious example is the work of Hubel and Wiesel, who investigated the electrophysiological response properties of single visual cortical cells to retinal input (1962, 1965), and later developed a theoretical framework for the functional division of primary visual cortex into hypercolumns (1977). This work led ultimately to a functional hierarchical model describing a divergence and re-integration of visual information proceeding from retinal ganglia, to the lateral geniculate nucleus of the thalamus, to primary visual cortex, and “upwards” to increasingly specialized and associative cortical regions.

While the notion of hierarchical organization appears straightforward for the first few relays proceeding from sensory neurons to primary cortex, even at these levels it is evident that information is segregated into components (e.g., spatial and temporal components), processed in parallel, reintegrated, integrated with cross-modal information, and fed backwards (Hubel and Wiesel, 1972; Rockland and Pandya, 1979; Livingstone and Hubel, 1988). Moreover, further refinement of this hierarchical theory added the notion of even higher levels capable of sending information signals “downward” to control the processing of incoming information and ultimately motor responses to it (Schulman *et al.*, 1997). As one proceeds along the processing pathway of sensory information, one encounters an increasing ambiguity with respect to the assignment of a functional hierarchy to the neuronal structures subserving this processing. To address this ambiguity, Felleman and Van Essen (1991) proposed that a simple anatomical criterion could be applied as a means of ordering information processing into hierarchical levels. Using connectivity information derived from tract

tracing experiments in macaque monkeys, and the idea that differential laminar source and termination patterns can distinguish ascending, descending, and lateral connections (Rockland and Pandya, 1979; Friedman, 1983; Maunsell and van Essen, 1983), the authors derived a hierarchical model (FV91) of the visual system including 32 cortical areas, describing 318 of the 992 pairwise hierarchical relationships between them.

One criticism of the FV91 model is that it is somewhat arbitrarily chosen from a model space which is on the order of  $10^{37}$  possible orderings. Hilgetag *et al.* (1996) examined this model space by assigning a cost function to a model's violation of the anatomical constraints imposed by experimentally-derived connectivity information, and report at least 150,000 optimal hierarchies, a number which is limited only by computational capacity. Each of these hierarchies requires six violations of the anatomical constraint set; the FV91 model, by comparison, has eight such violations. Moreover, the authors report that even if the model is constrained with a fully defined constraint set (i.e., assigning all unknown connections as existing), the number of equally optimal models remains large. Interestingly, the order position of the lowest cortical areas (V1 and V2) maintained their low ranking in all optimal models, whereas many of the higher areas varied greatly. While these findings appear to reinforce the idea of an inherent indeterminacy with respect to hierarchical organization in the brain, further investigation into the laminar patterns used to determine the direction of hierarchical projections suggests that a more refined constraint set is possible, which specifies a hierarchical "distance" in addition to relativity (Kennedy & Bullier, 1985; Barone et al, 2000; Batardière *et al.*, 2002; Grant & Hilgetag, 2005).

A further limitation of the hierarchical models presented above is their definition of hierarchical levels on a discrete scale, which carries the implicit assumption that the distance between levels is equal. This assumption necessitates multiple models to fit constraints that can be represented equally well by a single model specified using a more natural continuous hierarchical scale. Here we present a novel framework with which to define the hierarchical organization of cortical networks. Firstly, we define hierarchical position in terms of a continuous, rather than discrete, scale. Secondly, we alter the constraints on the cost function used to determine an optimal hierarchy by assigning ranges, rather than fixed values, to hierarchical relationships. Finally, where such information is available, we utilize the relative hierarchical distance information derived from laminar termination patterns to further refine the optimization.

## **Materials and Methods**

### ***Graph representation and the hierarchy function***

In order to obtain a hierarchical ordering in a cortical network we first represent it as a directed graph  $G = (V, E)$  consisting of a set of vertices  $V$  and a set of directed edges  $E$ . In the network each directed edge  $(u, v) \in E$  is assigned a weight describing the *hierarchical distance* from vertex  $u \in V$  to vertex  $v \in V$  (Figure 1A). The sign of this weight indicates the direction of this relationship (up or down) within the hierarchy. If the value of the edge is 0 then the edge does not cross levels in the hierarchy, i.e.  $u$  and  $v$  are on the same level. To determine the hierarchy implied by these edge values, we require a function  $h : V \rightarrow \mathbb{R}$  such that for every edge  $(u, v)$  with weight  $x$  the following condition holds:

$$h(u) + x = h(v). \tag{1}$$

This function  $h$  is called the *hierarchy function*.

A further possible refinement of this model is to expand our assumptions with respect to hierarchical relationships to allow a more flexible representation of distance. To accomplish this, edge weights can be replaced with continuous ranges of values such that, for every edge  $(u, v)$ , there is an interval  $[x, y]$ ,  $x, y \in \mathbb{R}$  (Figure 1B). This range expresses the interval in which the distance between  $u$  and  $v$  is presumed to lie. Such a representation requires a modification of our hierarchy function  $h$ :

$$h(u) + x \leq h(v) \text{ and } h(u) + y \geq h(v). \quad (2)$$

We can further modify  $h$  to accommodate uncertainty of our empirical data by including a deviation term  $\Delta$ . For an edge with a single weight  $x$  the hierarchy function becomes:

$$h(u) + x - \Delta \leq h(v) \text{ and } h(u) + x + \Delta \geq h(v). \quad (3)$$

For a range  $[x, y]$  this becomes:

$$h(u) + x - \Delta \leq h(v) \text{ and } h(u) + y + \Delta \geq h(v). \quad (4)$$

Inequalities (3) and (4) allow us to obtain an optimal hierarchy for  $G$ , in which  $\sum \Delta_i$ , the sum of  $\Delta_i$  over all edges  $i \in E$ , is minimal. To perform this optimization we use a well known method called linear programming (see, for instance, Papadimitriou and Steiglitz, 1998). In brief the aim of linear

programming is the optimization of a linear objective function, subject to linear equality and inequality constraints.

[Figure 1 about here]

### ***Empirical data***

Here we use the notation A+ for strongly ascending, A for ascending, L for lateral, D for descending and D+ for strongly descending assigned to each of the projections as reported in Table 5 of Felleman & Van Essen (1991). The optimization procedure was performed using the QSOPT Linear Programming Solver (2008), on two published data sets, and using a number of hierarchical relationship classification schemes, which allows comparison with existing literature describing cortical hierarchies. The first network considered was FV91, as described by Felleman & Van Essen (1991) for the visual system of the macaque monkey. As regions MDP and MIP had no constraints defined, they are not included here. The projections in this network were assigned ranges according to two different schemes. Firstly, ranges were assigned based upon the assumptions implied in the original FV91 article (see Figure 3, Felleman & Van Essen, 1991), which placed no effective outer bound on ascending or descending connections. Specifically, we used the ranges D: [-99, -1], L: 0, A: [1, 99], where 99 was chosen as an arbitrarily large outer bound. Secondly, a modification of the original relationship classification scheme was designed, which incorporates ideas presented in subsequent publications (Kennedy & Bullier, 1985; Barone *et al.*, 2000; Batardière *et al.*, 2002), and permits a richer representation of hierarchical distance and the assignment of refined ranges. This classification is illustrated in Figure 2. To investigate the effect of range sizes upon the resulting optimal hierarchies, ranges were systematically expanded by 0.1 at each limit, resulting in ten range

sets, as presented in Table 1. The outer bounds for these hierarchies were chosen as 32, which is the total number of regions; this ensures that no individual projection can have a hierarchical distance greater than the total number of regions.

[Table 1 about here]

[Figure 2 about here]

Barone *et al.* (2000) report that the proportion of retrogradely labelled neurons in the supragranular layers of visual cortical areas (referred to as SLN%) correlates strongly with the hierarchical rank (according to FV91) of the target structure. In other words, the authors suggest that the SLN% measure can be used as a real-valued hierarchical distance parameter specifying a more refined constraint on the cost function used to determine the optimal hierarchical model. We tested this possibility with a second visual network, obtained from Barone *et al.* (2000), making an initial assumption that SLN% values are directly comparable, and thus assigning these values directly as edge weights. The resulting hierarchy was obtained by optimizing according to the pair of inequalities in Eqn. 3.

## **Results**

### ***FVE original***

Figure 3 compares the hierarchy resulting from minimizing  $\Sigma\Delta_i$  in a visual network, under the connectivity classification presented in FV91, with the original hierarchy presented in that publication. Despite the use here of a continuous scale, the resulting optimal hierarchy is described



as discrete levels, similarly to FV91. This is likely accountable to the assignment of 0 to all lateral connections, and the assignment of ranges  $[-99, -1]$  and  $[1, 99]$  to non-lateral connections, ensuring a step of at least 1. While there appears to be a general correspondence between the two hierarchies, several differences are notable. AITd is placed alone at the top of new hierarchy, whereas in the original it is placed below the top level. V4 is placed on its own level in the new hierarchy, whereas it shares a level with other regions in FV91. Comparing the hierarchies numerically, we find that within the framework of our constraint set the resulting optimal hierarchy has  $\Sigma\Delta_i = 11$  and 9 total violations (having  $\Delta_i > 0$ ) of experimental data, whereas FV91 has  $\Sigma\Delta_i = 17$  and 12 total violations.

[Figure 3 about here]

### ***FVE modified***

Figure 4 illustrates the distributions of normalized optimal hierarchical level assignments for each cortical region in the FV91 network, across the ten variations of range parameters, as well as the sum of violations in normalized units across these ten variations as shown in Table 1. Without exception, V1 appears at the bottom and AITd at the top of every hierarchy generated. While the remaining regions display some variability in their optimal levels, Figure 4 shows a reasonably structured organization. Notably, some groups of regions appear to cluster at similar levels, which are visually distinguishable from other clusters.

[Figure 4 about here]

The distribution of these violations does not appear to be related to mean hierarchical level, and there is no significant correlation between these two variables ( $r = 0.0097$ ,  $t = 0.051$ ,  $p > 0.05$ ). Figure 5 illustrates the geometrical distribution of the mean hierarchy, both as three-dimensional cortical surfaces and a flat map representation. The latter also portrays the structural connectivity of the visual network.

[Figure 5 about here]

Figure 6 portrays the correlations of normalized hierarchical distances between cortical regions, across the ten constraint sets defined in Table 1. Given the fairly consistent ordering of regions within each hierarchy (Figure 4), all the correlations are quite high, but can be compared to get an indication of the relative stability of interregional relationships. Two clusters of regions are perfectly correlated. These clusters, {V3, VP} and {MT, V4, V4t}, have zero distance and zero variance, indicating that they always lie at precisely the same level. The distances between subdivisions of cortical region CIT (CITv and CITd) have a high correlation ( $r = 0.9997$ ), and relatively small mean distance (0.0018) and standard deviation (0.0054). Similarly, the subdivisions of region PIT (PITv and PITd) are highly correlated ( $r = 0.9976$ ) and have a relatively small mean distance (0.0054) and standard deviation (0.0161). By contrast, the two subregions of region AIT (AITv and AITd) now fall widely apart with  $r=0.9859$  with a mean distance 0.381 and standard deviation 0.0389.

[Figure 6 about here]

[Figure 7 about here]

***Barone et al.***

Figure 7 shows the hierarchy resulting from the optimization of the Barone *et al.* (2000) visual network, expressed in normalized arbitrary units. While the resulting hierarchy is similar to that reported earlier with these data (Vezoli *et al.*, 2004), a number of large discrepancies are apparent between the original SLN% values and the resulting hierarchical distances. These differences are largest for the uppermost regions of the network (THTF, FST, LIP, TE).

**Discussion**

We have presented a novel approach to the problem of obtaining an optimal hierarchy from a connected cortical network, based upon laminar origin and termination patterns. In particular, the use of a continuous hierarchical scale to replace the discrete levels used by both Felleman and Van Essen (1991) and Hilgetag *et al.* (1996) and definable ranges of acceptable hierarchical distances at least partially addresses the problem, introduced by Hilgetag *et al.*, of obtaining a unique optimal hierarchy for such a network. Under constraints analogous to those in Felleman and Van Essen (1991), we obtain an optimal hierarchy similar to that presented in the original paper, but having 9 constraint violations as opposed to 12. These values differ from those reported in Hilgetag *et al.* (1996), of 8 violations for the original hierarchy and 6 violations for each of its 150,000 optimal hierarchies, due to the different way we constructed the constraint sets. While the hierarchy reported here under these constraints is not necessarily unique, we note that the estimation of error along a continuous scale, rather than a discrete violation count, provides a more refined cost function.

The use of overlapping, continuous, real-valued ranges in place of discrete, integer-valued levels refines the representation of distance in hierarchical relationships, and also permits a systematic relaxation of the constraint sets used for optimization. This allows us to obtain a hierarchical distribution, shown in Figure 4, which is quite informative with respect to the robustness of optimal hierarchies across constraint sets. There is a clear hierarchical progression across the entire set of cortical regions, with relatively small regional variation, and there also appear to be clusters of regions which lie at approximately the same level. With respect to constraint violations, STPp, a subdivision of STP, emerges as the worst offender. Interestingly, STP has been postulated as a point of reconvergence of the dorsal and ventral visual streams (Young, 1992). It may prove informative to empirically investigate the reasons for such consistent violations by this and other regions, and how this affects the original assumptions relating laminar patterns to hierarchical relationships.

In a supplement to their 1996 paper, Hilgetag *et al.* make a number of predictions with respect to anatomical parcellation, based upon analysis of their optimal hierarchies. They predict that area FST may consist of two distinct subdivisions, since its hierarchical level was observed to be highly variable and its frequency distribution to be bi-modal. We indeed observe FST to be among the areas with frequent violations, but it does not appear to be anomalous within our optimized hierarchies, nor does it display exceptionally high variance (Figure 4). The authors further predict that four regional subdivisions are actually distinct areas, since their hierarchical distances are variable. We find this to be true in three cases, but in one case (PITv and PITd), we find a relatively high interregional distance correlation, and very small mean distance and variance. Another prediction, that V4t and MT are linked with one another, corresponds with our results, in which V4t, MT, and V4 are always placed at the same level. Likewise, CITd and CITv are very similar (Figure 6).

Interestingly, V3 and VP, a pair which is mentioned by Hilgetag *et al.* as one of particular interest for their potential analogy to dorsal and ventral hemi-regions, are also always placed at the same level in our hierarchies. The relationship between these regions was investigated subsequent to the 1996 article by Felleman *et al.* (1997) and collated in the CoCoMac database (Kötter, 2004). In this paper, the authors demonstrate that the subdivision of V3 and VP is justified by a number of observed differences in intracortical projection patterns. Our present results, as well as independent comparisons using multivariate analyses of neurotransmitter receptor distributions (Kötter *et al.*, 2001), suggest that, despite these differences, these two regions may share an identical hierarchical position. Regions V3A and PIP, on the other hand, whose anatomical distinction is also supported by the findings of Felleman *et al.*, are similarly found to be hierarchically distinct in the present study.

Reducing the scope of laminar distribution to that of cell counting permits a measurement of SLN%, whose relationship to hierarchical distance introduces a promising refinement of hierarchical optimization (Vezoli *et al.*, 2004, Grant and Hilgetag, 2005). Assigning SLN% directly as a value for hierarchical distance results in the normalized optimal hierarchy shown in Figure 7. While this hierarchy corresponds to that reported in Vezoli *et al.* (2004), several discrepancies are apparent. Particularly, distance values in this hierarchy can differ dramatically from the initial SLN% values, particularly for regions at the top of the hierarchy. While a degree of measurement error is expected in these data, it is unlikely that this error can account for such large deviations. Another possible explanation is that the assumption that SLN% values represent absolute hierarchical distance is inaccurate. It is more probable that these values represent a hierarchical distance that is relative to other projections within a particular tracer experiment, but is not directly comparable to SLN% values obtained from other experiments. This idea is supported by the observed discrepancy between

the only reciprocal connections in the network, V1 to V4 (SLN% = 1.0) and V4 to V1 (SLN% = -0.78). Such a problem might be addressed by optimizing across an entire network to fit the relative SLN% measurements for each region; however, this would require a more extensive data set than is presently available.

It is worth noting that the functional significance of anatomical hierarchies, such as those investigated here, remains unclear. Indeed, Hegdé and Felleman (2007) go so far as to suggest that such a representation is too overly simplistic to be useful as a basis for understanding the complexity of functional organization in the mammalian brain – which includes recurrent connectivity, temporal dynamics, contextual modulation, and multisensory inputs – and propose a Bayesian inference framework as a superior alternative. While these functional properties are probably real limitations of a hierarchical representation, we feel that such an anatomical blueprint still serves an important role in constraining the functional activity of a neural network, and may even be useful within a Bayesian framework. It is apparent from these observations, however, that in order to better capture the complexity of the brain, hierarchical models need to provide a more global representation of its anatomical network. Automated and refined optimization approaches such as the one presented here, providing for a real-valued hierarchical representation, may serve an important role in expanding the scope of hierarchical representations towards this end.

### **Information Sharing Statement**

We are glad to provide access to all data used in the present study. Please direct requests to the corresponding authors (AR, RK).

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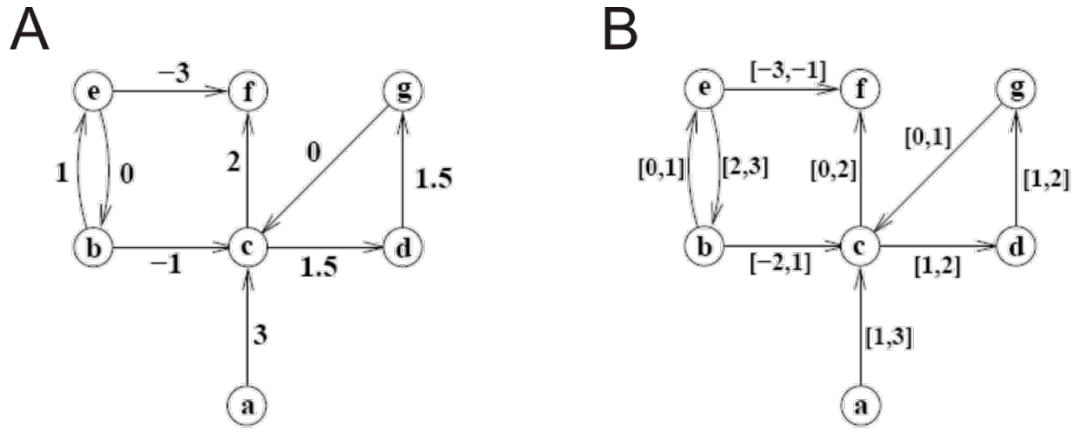
## References

- Barone P., Batardiere A., Knoblauch K., Giroud P., Berland M., Dumas A., Kennedy H., 2002. Early specification of the hierarchical organization of visual cortical areas in the macaque monkey. *Cereb. Cortex.* 12(5), 453-465.
- Barone P., Batardiere A., Knoblauch K., Kennedy H., 2000. Laminar distribution of neurons in extrastriate areas projecting to visual areas V1 and V4 correlates with the hierarchical rank and indicates the operation of a distance rule. *J. Neurosci.* 20(9), 3263-3281.
- Batardière A., Barone P., Knoblauch K., Giroud P., Berland M., Dumas A. M., Kennedy H., 2002. Early specification of the hierarchical organization of visual cortical areas in the macaque monkey. *Cereb. Cortex.* 12(5), 453-465.
- Dantzig G. B., 1963. *Linear Programming and Extensions.* Princeton University Press, Princeton, N. J.
- Felleman D. J., van Essen D. C., 1991. Distributed hierarchical processing in the primate. *Cereb. Cortex.* 1(1), 1-47.
- Felleman D. J., Burkhalter A., Van Essen D. C., 1997. Cortical connections of areas V3 and VP of macaque monkey extrastriate visual cortex. *J. Comp. Neurol.* 379(1), 21-47.
- Friedman D. P., 1983. Laminar patterns of termination of cortico-cortical afferents in the somatosensory system. *Brain Research.* 273(1), 147-151.
- Grant S., Hilgetag C. C., 2005. Graded classes of cortical projections: quantitative analyses of laminar projections to motion areas of cat extrastriate cortex. *European J. Neurosci.* 22, 681-696.
- Hegd  J., Felleman D. J., 2007. Reappraising the functional implications of the primate visual anatomical hierarchy. *Neuroscientist.* 13(5), 416-421.
- Hilgetag C. C., O'Neill M. A., Young M. P., 1996. Indeterminate organization of the visual system. *Science.* 271(5250), 776-777.

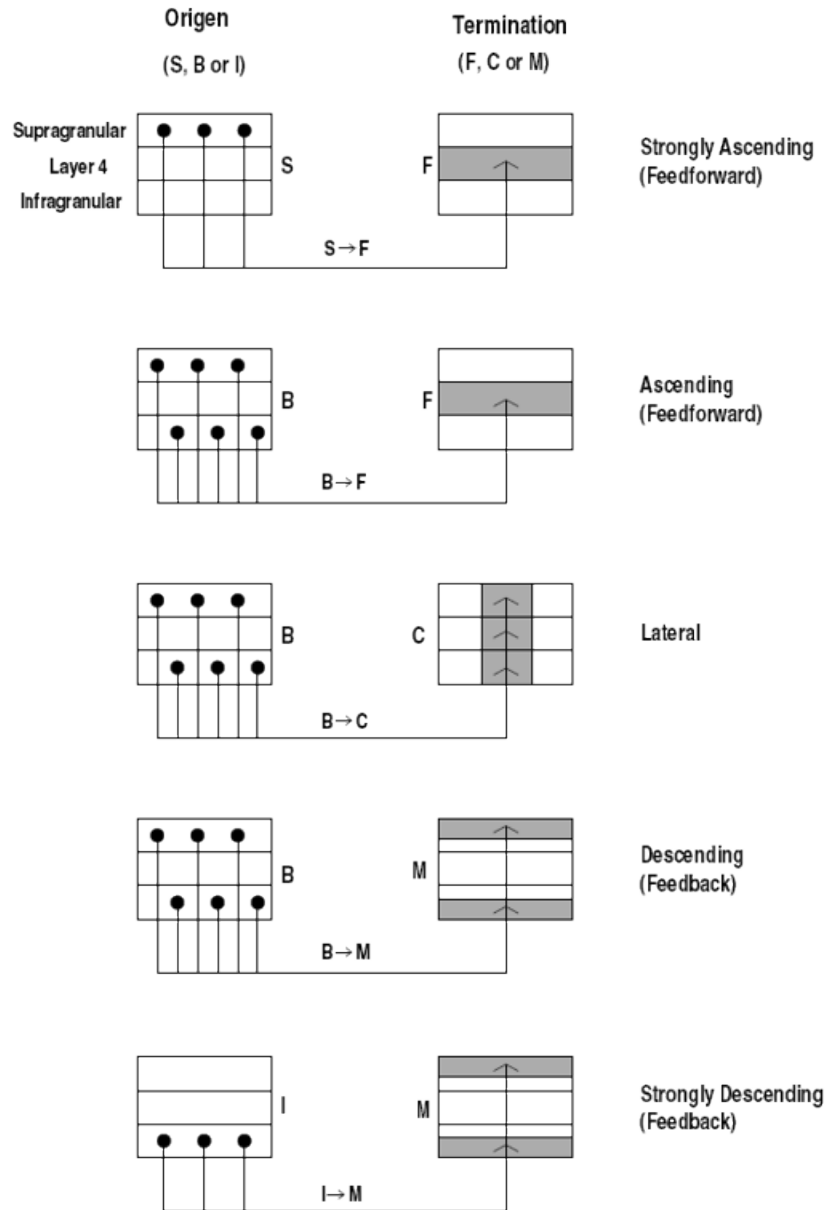


- Hubel D. H., Wiesel T. N., 1961. Receptive fields, binocular interaction and functional architecture in the cat's visual cortex. *J. Physiol.* 160, 106-154.
- Karmarkar N., 1984. A new polynomial-time algorithm for linear programming. *Combinatorica.* 4(4), 373-395.
- Kennedy H., Bullier J., 1985. A double-labeling investigation of the afferent connectivity to cortical areas V1 and V2 of the macaque monkey. *J. Neurosci.* 5(10), 2815-2830.
- Kötter R., 2004. Online retrieval, processing, and visualization of primate connectivity data from the CoCoMac database. *Neuroinformatics.* 2(2), 127-144.
- Kötter R., Stephan K.E., Palomero-Gallagher N., Geyer S., Schleicher A., Zilles K., 2001, Multimodal characterisation of cortical areas by multivariate analyses of receptor binding and connectivity data. *Anat. Embryol.* 204, 333-350.
- Livingstone M. S., Hubel D. H., 1988. Segregation of form, color, movement, and depth, anatomy, physiology, and perception. *Science.* 240(4853), 740-749.
- Maunsell J. H., van Essen D. C., 1983. The connections of the middle temporal visual area, MT) and their relationship to a cortical hierarchy in the macaque monkey. *J. Neurosci.* 3(12), 2563-2586.
- Papadimitriou C. H., Steiglitz K., 1998. *Combinatorial optimization, algorithms and complexity.* Mineola., NY, Dover Publications, Inc.
- QSOPT Linear Programming Solver. 15 Sept., 2008. <<http://www.isye.gatech.edu/~wcook/qsopt/>>.
- Rockland K. S., Pandya D. N., 1979. Laminar origins and terminations of cortical connections of the occipital lobe in the rhesus monkey. *Brain Res.* 179(1), 3-20.
- Shulman G. L., Corbetta M., Buckner R. L., Raichle M. E., Fiez J. A., Miezin F. M., Petersen S. E., 1997. Top-down modulation of early sensory cortex. *Cereb. Cortex.* 7(3), 193-206.
- Vezoli J., Falchier A., Jouve B., Knoblauch K., Young M., Kennedy H., 2004. Quantitative analysis of connectivity in the visual cortex, extracting function from structure. *Neuroscientist.* 10(5), 476-482.

**Figures and Tables**



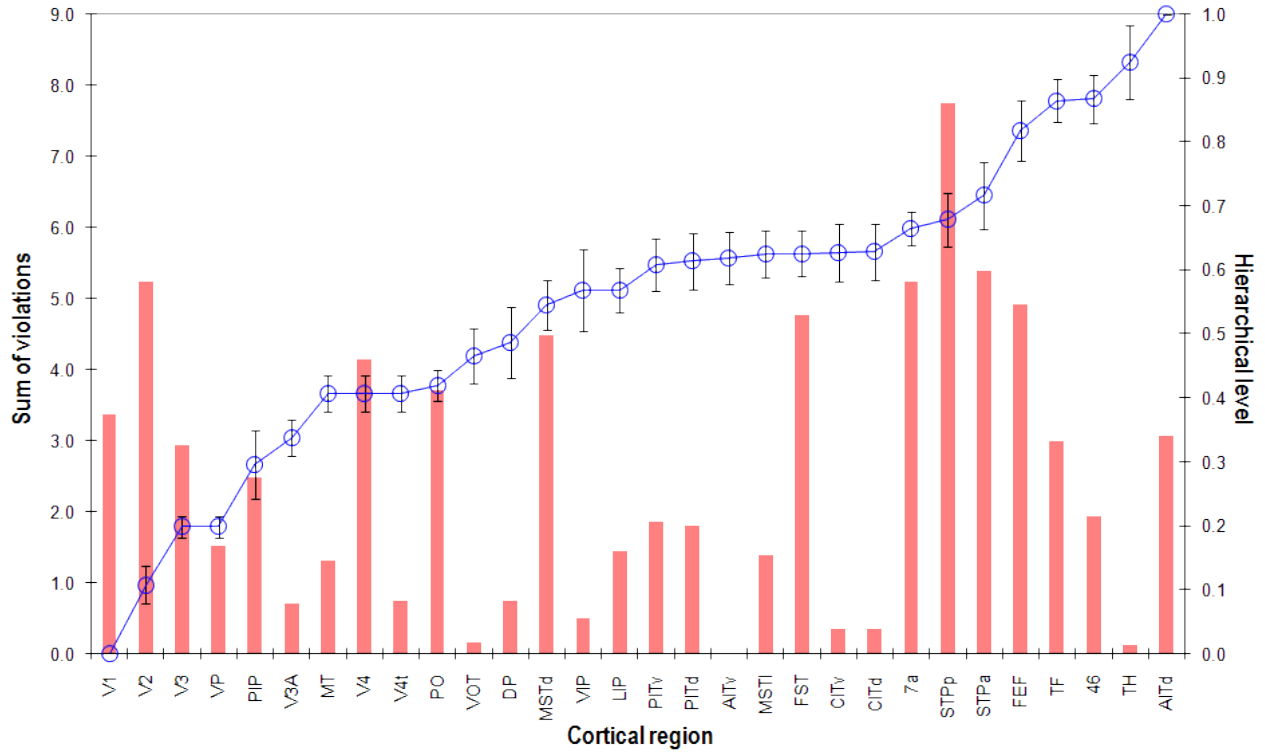
**Figure 1.** Examples of cortical networks represented as graphs with (A) real-valued discrete edge weights representing hierarchical distance and (B) real-valued ranges representing a presumed range of hierarchical distances.



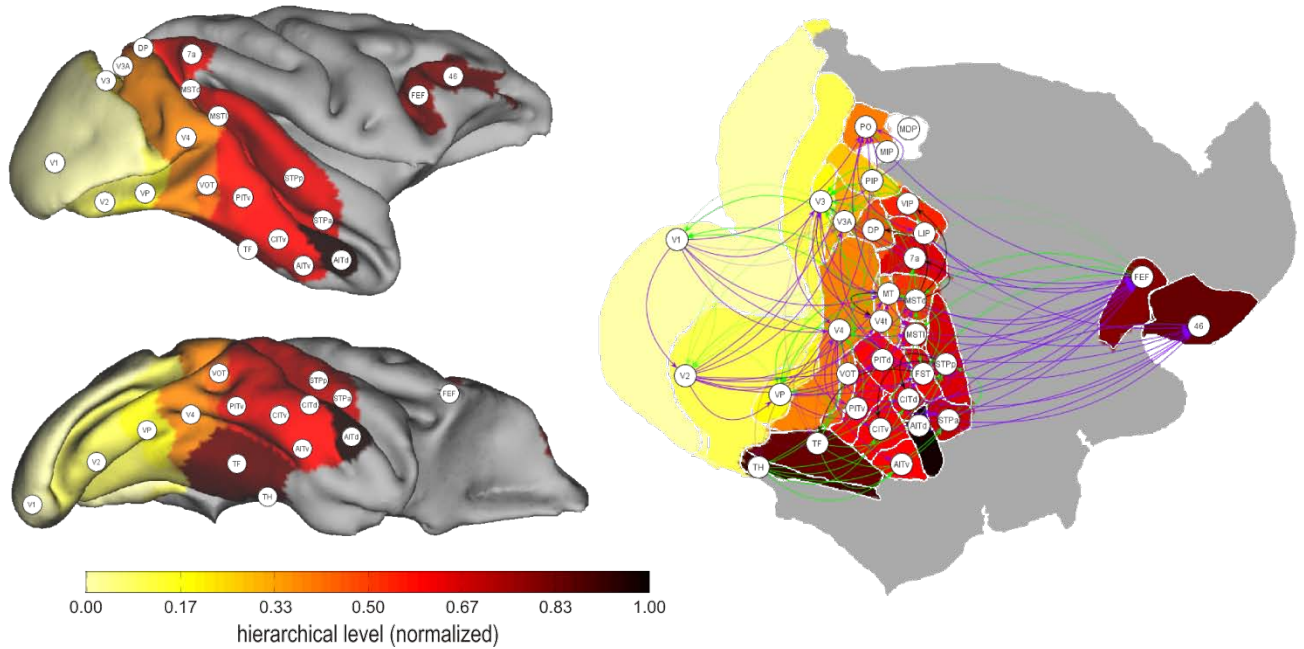
**Figure 2.** Refined classification scheme of hierarchical relationships for the FV91 network, which permits a representation of hierarchical distance.

	Minimizing $\sum \Delta_i$	Hierarchy from [FE91]
$\sum \Delta_i$ :	11	17
number of constraints with $\Delta_i > 0$ :	9	12
	Hierarchy levels	
	AITd	10
	46 TH	9
	FEF TF STPa	8
	7a STPp CITd CITv	7
	DP VIP LIP MSTd MSTl FST PITd PITv	6
	AITv PO MT VOT V4t	5
	V4	4
	PIP V3A	3
	V3 VP	2
	V2	1
	V1	0
	46 TF TH	46
	STPa AITd AITv	7a
	7a FEF STPp CITd CITv	VIP LIP MSTd MSTl FST PITd PITv
	DP VOT	MDP MIP PO MT V4t V4
	PIP V3A	V3 VP
	V3 VP	V2
	V2	V1

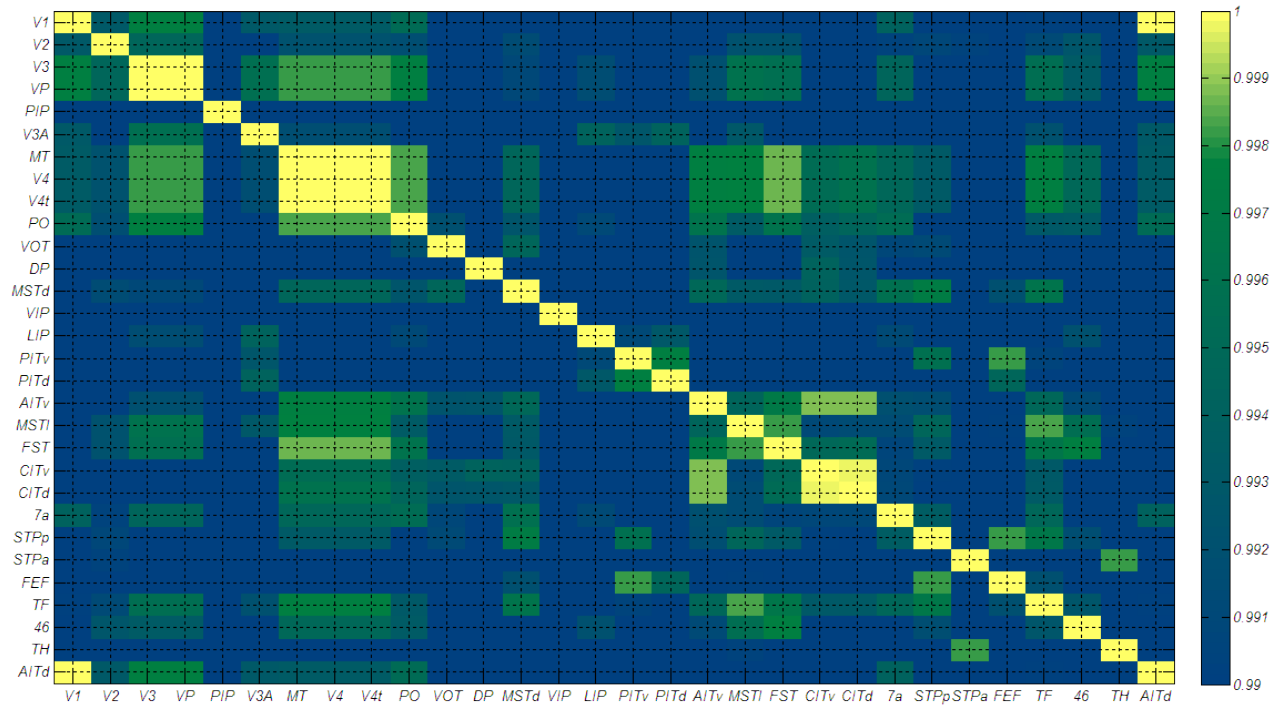
**Figure 3.** Comparison of original hierarchy from FV91 (right) with that obtained under the same implicit constraints, by minimizing  $\sum \Delta_i$  (left).



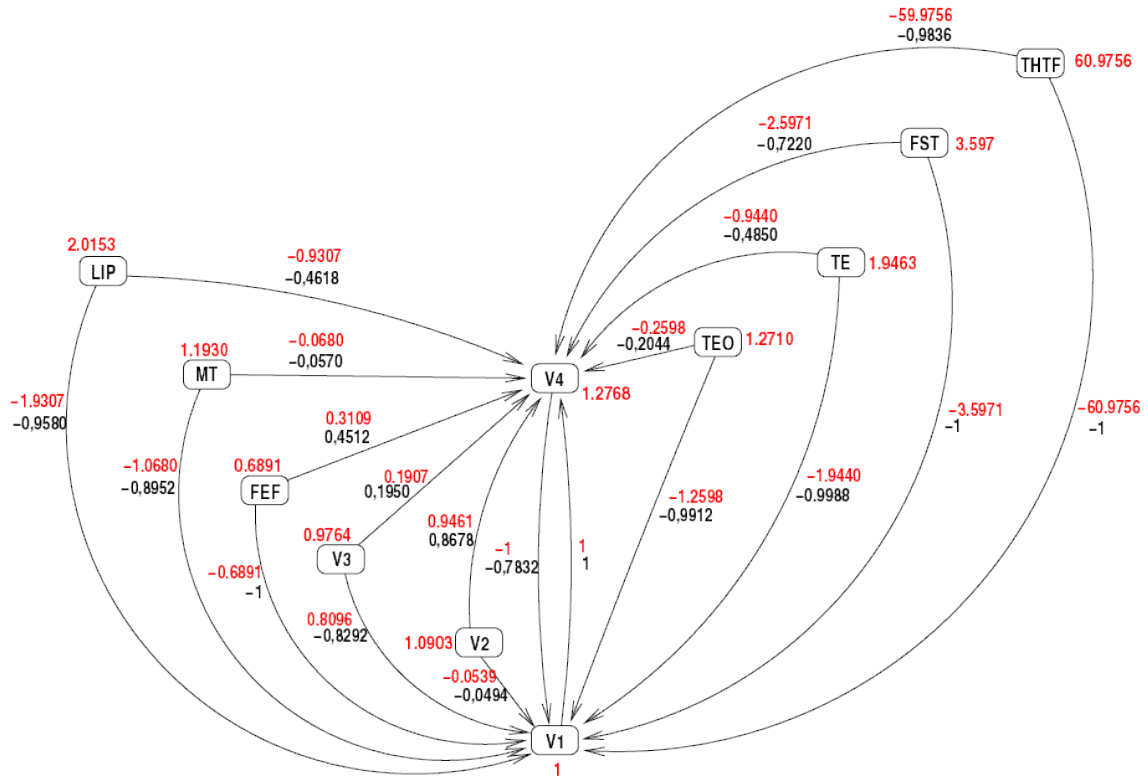
**Figure 4.** Hierarchical level and violations are expressed as arbitrary units, normalized within each hierarchy to the range of that hierarchy. *Blue circles:* Mean hierarchical level for each cortical region in the FV91 network, across ten range sets. Error bars represent standard deviation. *Pink bars:* Sum of violations for each region across these ten variations.



**Figure 5.** Geometrical distribution of hierarchy levels in the FV91 visual network of the macaque, both as three-dimensional cortical surface renderings (left), and as a two-dimensional “flat map” representation of the cortical sheet (right). Regions are coloured by their mean normalized hierarchical position, obtained over the ten constraint sets described in Table 1. Directed edges in the flat map illustration represent interregional connections, and are coloured according to projection class: A+ (purple, opaque), A (purple, transparent), L (black), D (green, transparent), and D+ (green, opaque). Compare this representation to figs. 2 and 4 in Felleman and Van Essen, 1991.



**Figure 6.** Pseudocolour correlation map of interregional hierarchical distance across the ten constraint sets defined in Table 1. Regions are sorted by their mean hierarchical level (compare to Fig. 4).



**Figure 7.** The hierarchy obtained from the Barone *et al.* (2000) data set, expressed in normalized arbitrary units. Black numbers indicate the original SLN% values, red numbers indicate distance and position within the optimal hierarchy (where they differ from SLN%).

Class	Set 0		Set 1		Set 2		Set 3		Set 4		Set 5		Set 6		Set 7		Set 8		Set 9	
	From	To	From	To	From	To	From	To	From	To	From	To	From	To	From	To	From	To	From	To
A+	-32	-2	-32	-1.9	-32	-1.8	-32	-1.7	-32	-1.6	-32	-1.5	-32	-1.4	-32	-1.3	-32	-1.2	-32	-1.1
A	-1	-1	-1.1	-0.9	-1.2	-0.8	-1.3	-0.7	-1.4	-0.6	-1.5	-0.5	-1.6	-0.4	-1.7	-0.3	-1.8	-0.2	-1.9	-0.1
L	0	0	-0.1	0.1	-0.2	0.2	-0.3	0.3	-0.4	0.4	-0.5	0.5	-0.6	0.6	-0.7	0.7	-0.8	0.8	-0.9	0.9
D	1	1	0.9	1.1	0.8	1.2	0.7	1.3	0.6	1.4	0.5	1.5	0.4	1.6	0.3	1.7	0.2	1.8	0.1	1.9
D+	2	32	1.9	32	1.8	32	1.7	32	1.6	32	1.5	32	1.4	32	1.3	32	1.2	32	1.1	32

**Table 1:** Ranges used for optimization of the refined classification scheme of FV91.

Ten value ranges were used as constraint sets for the optimization. Outer limits were kept at comparatively large values (32, the total number of cortical regions) while inner limits were systematically expanded by increments of 0.1.