The Auditory System and Human Sound-Localization Behavior Short Answers Exercises Chapter 13: Multisensory Integration.

Exercise 13.1:

The race model predicts that the reaction time to an audiovisual stimulus is determined by the minimum delay of the visual and auditory modalities. The distribution of AV reaction times is therefore given by the distribution of minima. This distribution is composed of those reactions that are triggered by the auditory stimuli (when the auditory input reached the decision at time τ first) plus the distribution of responses that were triggered by the visual stimulus (when the visual input reached the reaction criterion first). Because the visual and the auditory modalities are processed independently, the probability that the auditory stimulus triggered the response at time τ is given by $A(\tau)$ multiplied by the cumulative probability that the visual input reached the criterion at τ *later* than auditory:

$$R_{AV}(\tau, A) = A(\tau) \cdot \int_{\tau}^{\infty} V(s)ds$$

Likewise, the probability that the visual stimulus triggered the AV response is given by the cumulative probability that the auditory input reached the criterion later than visual at that time:

$$R_{AV}(\tau, V) = V(\tau) \cdot \int_{\tau}^{\infty} A(s)ds$$

The distribution of minimum reaction times for independent auditory and visual channels is therefore the sum:

$$R_{AV}(\tau) = A(\tau) \cdot \int_{\tau}^{\infty} V(s)ds + V(\tau) \cdot \int_{\tau}^{\infty} A(s)ds$$

Exercise 13.2:

Product of two Gaussians:

Suppose, two Gaussians, say one for visual and the other for auditory, which are given by:

$$G_V(x) = N_V \cdot \exp - \frac{(x-\mu_V)^2}{S_V} \quad \text{en} \quad G_A(x) = N_A \cdot \exp - \frac{(x-\mu_A)^2}{S_A}$$

If both distributions are independent, the joint probability is given by their product:

$$G_{AV}(x) = N_{AV} \cdot \exp - \frac{(x-\mu_V)^2}{S_V} \cdot \exp - \frac{(x-\mu_A)^2}{S_A}$$

$$= N_{AV} \cdot \exp\left(-x^2 \cdot \left(\frac{1}{S_V} + \frac{1}{S_A}\right) - 2x \cdot \left(\frac{\mu_V}{S_V} + \frac{\mu_A}{S_A}\right) + \left(\frac{\mu_V^2}{S_V} + \frac{\mu_A^2}{S_A}\right)\right)$$

Rewrite the exponent by using

$$ax^2 + bx + c = a \cdot (x + \frac{b}{2a})^2 + (c - \frac{b^2}{4a})$$

You will then obtain a new Gaussian: with variance given by

$$\sigma_{AV}^2 = \frac{S_A \cdot S_V}{S_A + S_V} = \frac{\sigma_A^2 \cdot \sigma_V^2}{\sigma_A^2 + \sigma_V^2}$$

and mean:

$$\mu_{AV} = \frac{S_A \mu_V + S_V \mu_A}{S_A + S_V} = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_V^2} \cdot \mu_V + \frac{\sigma_V^2}{\sigma_A^2 + \sigma_V^2} \cdot \mu_A$$

Note that the bimodal estimate always falls *between* the two uni-modal estimates (*'averaging'!*), and that the precision is always *better* ithan the precision of *best* unimodal stimulus!

The normalized joint distribution thus becomes

$$G_{AV}(x) = \frac{1}{\sigma_{AV}\sqrt{2\pi}} \cdot \exp - \left(\frac{(x - \mu_{AV})^2}{2\sigma_{AV}^2} \right)$$

Exercise 13.3:

Product of three Gaussians: this is a continuation of the previous exercise. So, now you consider the product of G_{AV} with G_P, with

$$G_{AV} = N_{AV} \cdot \exp\left(-\frac{(x - \mu_{AV})^2}{2\sigma_{AV}^2} \right) \quad \text{and} \quad G_P = N_P \cdot \exp\left(-\frac{(x - \mu_P)^2}{2\sigma_P^2} \right)$$

and from the previous exercise we immediately obtain

$$G_{AVP} = N_{AVP} \cdot \exp\left(-\frac{(x - \mu_{AVP})^2}{2\sigma_{AVP}^2} \right)$$

in which

$$\sigma_{AVP}^2 = \frac{\sigma_{AV}^2 \sigma_P^2}{\sigma_{AV}^2 + \sigma_P^2} \quad \text{and} \quad \mu_{AVP} = \frac{\sigma_{AV}^2}{\sigma_{AV}^2 + \sigma_P^2} \cdot \mu_P + \frac{\sigma_P^2}{\sigma_{AV}^2 + \sigma_{AV}^2} \cdot \mu_{AV}$$

with σ_{AV} and μ_{AV} from the frevious exercise. Combining yields:

$$\sigma_{AVP}^2 = \frac{\sigma_A^2 \sigma_V^2 \sigma_P^2}{\sigma_A^2 \sigma_V^2 + \sigma_A^2 \sigma_P^2 + \sigma_V^2 \sigma_P^2} \quad \text{and} \quad \mu_{AVP} = \frac{\sigma_A^2 \sigma_V^2 \cdot \mu_P + \sigma_V^2 \sigma_P^2 \cdot \mu_A + \sigma_A^2 \sigma_P^2 \cdot \mu_V}{\sigma_A^2 \sigma_V^2 + \sigma_A^2 \sigma_P^2 + \sigma_V^2 \sigma_P^2}$$

Exercise 13.4:

This exercise refers to the analysis of Eqn. 13.20, in case mean and variance are not determined by the MAP estimates (Eqn. 3.19), but by the posterior distribution itself. The mean and variance of the posterior are then given by

$$\mu_{R,Az} = \frac{S_P}{S_P + S_{Az}} \cdot \mu_{Az} \equiv G_{Az} \mu_{Az} \quad \text{and} \quad S_{R,Az} = \frac{S_P S_{Az}}{S_P + S_{Az}}$$

$$\mu_{R,El} = \frac{S_P}{S_P + S_{El}} \cdot \mu_{El} \equiv G_{El} \mu_{El} \quad \text{and} \quad S_{R,El} = \frac{S_P S_{El}}{S_P + S_{El}}$$

So if we calculate the ratio's for the gains we find the same as for the MAP model:

$$\frac{G_{El}}{G_{Az}} = \frac{S_P + S_{Az}}{S_P + S_{El}} < 1 \text{ because } S_{El} > S_{Az}$$

and

$$\frac{S_{R,El}}{S_{R,Az}} = \frac{S_{El} \cdot (S_P + S_{Az})}{S_{Az} \cdot (S_P + S_{El})} = \frac{S_{El} + \frac{S_{El} S_{Az}}{S_P}}{S_{Az} + \frac{S_{El} S_{Az}}{S_P}} > 1$$

Indeed, in case the MAA in elevation (2.5 deg) and azimuth (0.7 deg) around straight ahead are taken as estimates for the noise in these components, the ratio for the variance in localization directions is about 3.5, which is comparable to measured response data.

Exercise 13.5:

The response gain and response variance are given by

$$G = \frac{S_P}{S_P + S_T} \text{ and } S_R = \frac{S_P S_T}{S_P + S_T}$$

with S_P the variance of the prior, and S_T the variance of the likelihood function. Combining these two expressions yields

$$1 - \frac{S_R}{S_P} = 1 - \frac{S_T}{S_P + S_T} = \frac{S_P}{S_P + S_T} \equiv G \text{ q.e.d}$$

Exercise 13.6:

Eqn. 13.21 tells us that the relationship between response variance and response gain holds for all response components and all acoustic conditions! The straight line interesects the $G = 0$ at the variance of the prior. So let's estimate some of these gain-variance combinations from Fig. 13.4, where we notice that the ellipses are drawn at 2σ:

Stimulus	**Gain**	**Std. Dev. (deg)**	**Variance**	σ_P (deg)
A6	0.7	5.5	30	10
A12	0.45	7.5	56	10
A18	0.15	9	81	9.8
A21	0.12	11	121	11.7

The estimated prior standard deviation is about 10-11 deg, which seems to be a reasonable estimate for a straight-ahead prior. Using this result, it is now also possible to predict the variances in the azimuth data through

$$S_{R,Az} = S_P \cdot (1 - G_{Az})$$

For example, at a gain of 0.8 the expected variance is about 100 x 0.2 = 20 deg^2 ($\sim$4.5 deg), which seems to be a reasonable estimate. At a gain of 0.5 the variance increases to 100 x 0.5 = 50 deg^2 (7 deg).

Exercise 13.7:

This exercise refers to the data of Fig. 13.12 (page 382). We extend Table 13.1 of paget 382 with the -12 dB and -18 dB sounds:

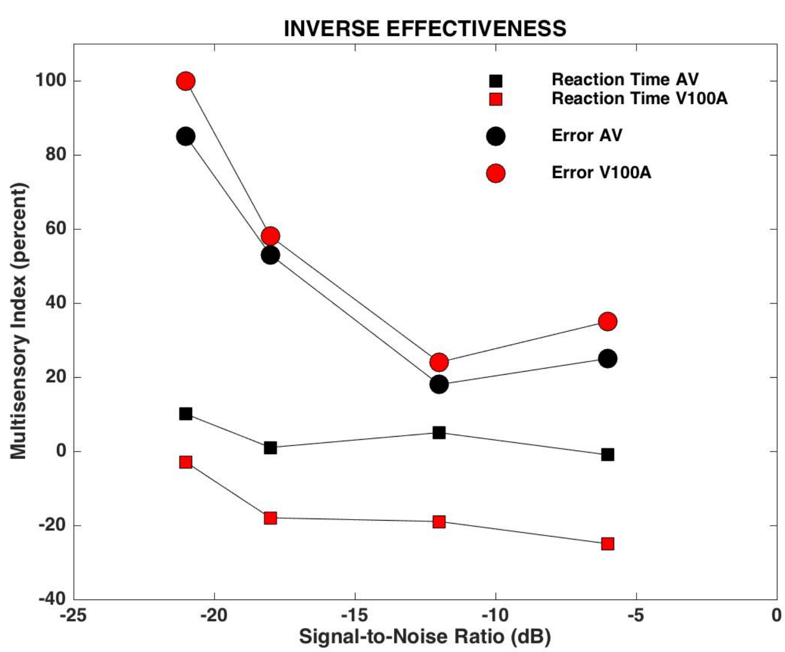
	A-only		AV		V100A	
SNR	ERR	RT	ERR	RT	ERR	RT
-6 dB	1.05	0.55	0.80	0.56	0.70	0.80
-12 dB	0.95	0.63	0.77	0.58	0.71	0.82
-18 dB	1.35	0.68	0.82	0.67	0.77	0.86
-21 dB	1.80	0.90	0.95	0.80	0.80	0.93

From these data we compute the Multisensory Indices using Eqn. 13.27, yielding:

$$MI = (R_A - R_{AV}) \cdot 100\%$$

Stim	MI Reaction Times (%)	MI Errors (%)
AV_6	-1	+25
$V100A_6$	-25	+35
AV_{12}	+5	+18
$V100A_{12}$	-19	+24
AV_{18}	+1	+53
$V100A_{18}$	-18	+58
AV_{21}	+10	+85
$V100A_{21}$	-3	+100

The figure nicely shows the Inverse Effectiveness of Multisensory Integration:



Inverse effectiveness data from the table. The effect is indeed strongest for the lowest signal-to-noise ratio's. The effect is weaker on the reaction times.