The Auditory System and Human Sound-Localization Behavior

Exercises Chapter 10

Problem 10-1 Show that the following mapping function also satisfies the constraint of Eqn. 10.2:

$$T(r) \equiv r' = \sqrt{N_0^2 + \frac{2N}{\pi \cdot \alpha} \cdot \ln \frac{r}{r_0}} \text{ and } T(\phi) \equiv \phi' = \phi$$

with N_0 and r_0 arbitrary constants, and α the proportionality constant for receptive field size. Plot the iso-eccentricity and iso-direction lines in a 2D representation of this function.

Problem 10-2

a Draw a graph of the shifted complex-log function of Eqn. 10.6. Calculate and plot the images of the vertical meridians, and of iso-direction lines at ±30 and ±60 deg, as well as for a number of logarithmically spaced eccentricities. Is this map conformal? Why (not)?

b Provide a prescription for the inverse mapping of this function:

$$T^{-1}(u, v) = (r, \phi)$$

Problem 10-3 Rewrite the SC afferent mapping function to *(x,y)* coordinates.

Problem 10-4 Verify the expression for the inverse mapping function of Eqn. 10.11 (the efferent map) that relates the horizontal (*x*), and vertical (*y*) saccade components ($z_O = (x,y)$) to collicular neural coordinates, $w_O = (u_O, v_O)$. Also express the efferent map in polar coordinates.

Problem 10-5 If the cell density in the SC motor map is taken constant, at ρ_0 cells/mm^2, show that the total number of spikes from the SC population is given by

$$N_{TOT} = 2\pi \cdot N_0 \cdot \rho_0 \cdot \sigma_{POP}^2$$

Make an educated guess for N_{TOT}.

Problem 10-6 Use Eqn. 10.12 to demonstrate that the width of the movement field (determined by the efferent mapping of 1.0 mm of SC space, symmetrically positioned around the center of a population at u_0) increases linearly with R_0. Also determine the asymmetry of the MF by taking the ratio between low-edge to peak vs. peak-to-high edge.

Problem 10-7 Suppose that the center of the active Gaussian cell population in the SC determines the size and direction of the saccade vector endpoint. Assume that center location of the population is endowed with some noise, due to retinal uncertainty, so that will scatter from saccade to saccade in response to identical target presentations. Suppose that the SC scatter is bounded by a small *circular* region around the true center, with radius ε in the collicular complex-log map of Eqn. 10.9. Derive an expression for the distribution of saccade vectors as function of saccade amplitude that results from this scatter, and show that under these assumptions the anisotropy of the motor map is directly reflected in the saccade endpoint distributions.

Hint: Place the scatter with its center on the horizontal meridian of the map and compute the resulting vectors for 5 points on this circle: the center, the horizontal meridian intersection points of the circle, and the two most up/down vertical points on the circle. These 5 points define the long and short axes of an elliptical distribution.

Problem 10-8 Since the horizontal/vertical brainstem feedback circuits (with gain B, and feedback delay, ΔT) and the downstream PSGs of Fig. 10.6 are all *linear* systems, their total function can be replaced by single feed-forward models with identical input-output characteristics.
Derive these characteristics, assuming simple, first-order plant models (time constant, T, and add a gain T in the direct path from PGs to motor neurons).

Problem 10-9 Explain the deficits observed in the simulated saccade vectors with the model in Fig. 10.10A.