The Auditory System and Human Sound-Localization Behavior

Exercises Chapter 2

Problem 2-1: Verify Eqn. 2.23 for the speed of gas molecules under adiabatic conditions.

Problem 2-2:

(a) Show that the homogeneous wave equation, Eqn. 2.25, indeed holds for *any* function, $s(x,t)$, that can be written as:

$$s(x,t) = s(x \pm vt)$$

This is a very important and fundamental result as it states that any wave shape that travels at a constant velocity through the medium obeys the homogeneous wave equation (and it is a non-dispersive medium).

(b) Verify the linearity condition (superposition of solutions) of the homogeneous wave equation.

Problem 2-3: Show Eqn. 2.30 by substituting the harmonic traveling wave function after separation of variables.

Problem 2-4: Demonstrate that the standing-waves of the fixed boundary conditions for $s(0,t) = s(L_0,t)$ lead to the solutions given by Eqn. 2.36.

Problem 2-5:

(a) Follow a similar analysis as you did in Problem 2-4 to find the standing waves under open boundary conditions: $s(0,t)=$ and $s(L_0,t)=$ maximum.
(b) Same for mixed boundary conditions: $s(0,t)=0$ and $s(L_0,t)=$ maximum.
(c) Same for periodic boundary conditions: $s(0,t)=s(L_0,t)$ and $\frac{\partial s}{\partial x}(0,t) = \frac{\partial s}{\partial x}(L_0,t)$.

Problem 2-6: The inhomogeneous wave equation, Eqn. 2.50, has harmonic temporal solutions, but nonharmonic spatial solutions. Demonstrate this latter statement.

* **Problem 2-7:** In some cases, the inhomogeneous wave equation, Eqn. 2.50, can be solved analytically. Consider *fixed* boundary conditions at $x=a$ and $x=2a$, and substitute for the spatial dependence of the density and bulk modulus:

$$\rho(x) = \frac{m}{x^2}, \qquad B(x) = B_0$$

Demonstrate that the standing waves described by Eqn. 2.51 (and illustrated in Fig. 2.7) are indeed a solution of the inhomogeneous wave equation, Eqn. 2.50.

* **Problem 2-8:** A second example of an inhomogeneous standing wave problem is the following: Consider a string of length L with mass density ρ_1 and tension B, that is connected to a second string of the same length and tension, but with mass density ρ_2. Both strings are attached to walls that are $2L$ apart, so that standing waves will arise for fixed boundary conditions (Figure 2.11).

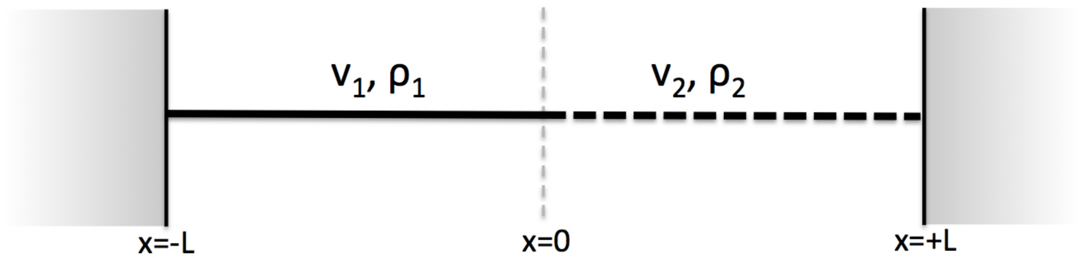

Figure 2.11 *Inhomogeneous string, consisting of two equal-length (L) parts with different mass densities, attached in the center at x=0, and fixed boundary conditions at x=-L and x=+L.*

(a) Show that the frequencies of the eigenmodes of this system obey the following nonlinear relation:

$$v_1 \tan\left(\frac{\omega L}{v_1}\right) = -v_2 \tan\left(\frac{\omega L}{v_2}\right)$$

(b) Find an expression for the standing wave solutions, *s(x,t)*

(c) Is it possible that $x=0$ is a node? If so, when?

Problem 2-9: Show that the total kinetic energy of a harmonic sound wave, described by $s(t) = s_{\max} \cdot \cos(\omega t)$, confined to a gas cylinder with cross section A, taken over a full wavelength, λ, is given by Eqn. 2.52:

$$K_\lambda = \frac{\rho}{4} \cdot A \cdot (\omega \cdot s_{\max})^2 \cdot \lambda$$

Problem 2-10: Verify the transmitted and reflected intensities, I_T and I_R, respectively, at the boundary of two media with acoustic impedances Z_1 and Z_2, respectively (see Eqn. 2.52)

*** Problem 2-11:**

Show that sine and cosine obey the so-called orthogonality relations:

$$\int_0^T \cos(n\omega t) \cdot \cos(m\omega t) = \int_0^T \sin(n\omega t) \cdot \sin(m\omega t) = \frac{T}{2} \cdot \delta_{nm}$$

$$\text{and} \int_0^T \sin(n\omega t) \cdot \cos(m\omega t) = 0$$

with T the period that is related to the angular frequency by $\omega = \frac{2\pi}{T}$

*** Problem 2-12:** Using the orthogonality relations of the previous exercise, you can now demonstrate the validity of Eqns. 2.68 (the calculation of the discrete Fourier spectrum).

Hint: to calculate a_0, take the time-average of the left- and right-hand sides of Eqn. 2.67. To determine the coefficients a_n, b_n multiply both sides of Eqn. 2.67 with $cos(m\omega t)$ and $sin(m\omega t)$, respectively, and take the time-overage.

Problem 2-13: A Fourier series for a periodic function without boundary ~ and initial conditions:

(a) Determine the Fourier series for $f(t)=t^2-t$, on the interval $0 \leq t \leq 1$: expand the function such that it becomes an *odd* periodic function.

(b) Idem for the case where the function of (a) is made *even*.

(c) Approximate f(x) by the first three Fourier components of the series in (a) and (b). Compare the predicted values of both series with the actual values of the function in t=[0.0, 0.2, 0.4, 0.6, 0.8, 1.0] and draw the results. Which of the two Fourier series converges fastest to $f(t)$? Why?

Problem 2-14: Consider the following function, $f(t)$ on the interval $0 \leq t \leq 1$:

$$f(t) = \begin{cases} 1 - ct \ \text{ for } \ 0 \leq t \leq \frac{1}{c} \\ 0 \ \text{ for } \ \frac{1}{c} \leq t \leq 1 \end{cases} \qquad c \geq 1 \text{ is a constant}$$

(a) Expand $f(t)$ to an even function with period T=2. Draw this function.

(b) Write $f(t)$ as a Fourier series and determine the Fourier coefficients.

(c) Explain what happens to the spectrum of $f(t)$ as $c \to \infty$

Problem 2-15: A Fourier problem with boundary and initial conditions. Consider a string that has a length of 5π, which is fastened at x=0 and x=5π. The string obeys the following wave equation:

$$\frac{\partial^2 y}{\partial t^2} = 25 \cdot \frac{\partial^2 y}{\partial x^2}$$

with $y(x,t)$ the transversal deflection of the string from equilibrium over the interval $x \in [0,5\pi]$. The following initial conditions hold for t=0:

$$y(x,0) = \sin(x) \cdot (1 + 2\cos(x)) \ \text{ and } \ \dot{y}(x,0) = 0 \ \ \forall x$$

Determine $y(x,t)$