The Auditory System and Human Sound-Localization Behavior

Exercises Chapter 5

Problem 5.1 (a) Show that Eqn. 5.12 is indeed a solution that obeys the dry-water constraints Eqns. 5.9 and 5.10

(b) Make the deep-water wave approximation to show Eqn. 5.13

(c) Same for the shallow-water wave approximation of Eqn. 5.14

Problem 5.2 (a) Derive the dispersion relation for gravitational water waves, Eqn. 5.17

(b) Compute the phase velocity for deep-water waves. What is the group velocity?

(c) The same for shallow-water waves.

Problem 5.3 (a) Apply the combined effects of gravity and tension to determine the total dispersion relation of dry water, Eqn. 5.21.

(b) Determine the phase- and group velocity.

(c) Show that there is one particular wave length, for which phase- and group velocities are identical (which wave length? How high is this velocity?).

Problem 5.4 Consider an infinite basin of dry water, stretching from $x \in [-\infty, +\infty]$. Ignore the *z*-dimension. For $x<0$ the depth of the basin is $h=h_1$, so that $y_{\text{bottom}}=-h_1$. At $x=0$ the bottom profile suddenly jumps upward to $h=h_2<h_1$. A traveling water wave comes from the left ($x<0$) at amplitude A, and moves rightward.

We consider the deep-water case, i.e. $\lambda \ll h_{1,2}$.

(a) Compute the reflection, R, and transmission, T, at x=0 (use: *impedance*).

(b) Same for the shallow-water case, $\lambda \gg h_{1,2}$.

Problem 5.5 Negative dynamic feedback with a delay can cause instability problems. In this exercise we analyse the influence of a delay on the transfer characteristic of a linear system with feedback (Figs. 5.25 and 5.26).

(a) Determine the Laplace transform of a pure delay: $y(t) = x(t-\Delta T)$, and from that calculate its transfer characteristic in the frequency domain.

(b) Consider the system shown in Fig. 5.25. Determine the total transfer function and the loop gain. The system will spontaneously oscillate, and thus become unstable, when the loop-gain exceeds the value of 1, and at the same time has a phase shift of -180°. Perform a Bode analysis on this system and estimate the frequency ω_0 where instability kicks in (Fig. 5.26).

(c) What happens to the system if A is increased/lowered? What if the time constant T is increased/lowered?

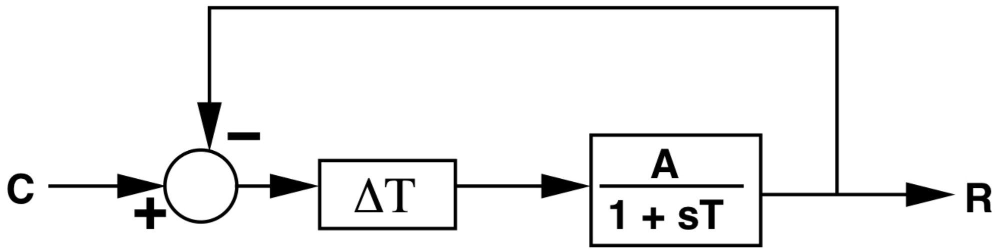

Figure 5.25 *Dynamic feedback model of a low-pass filter with a delay.*

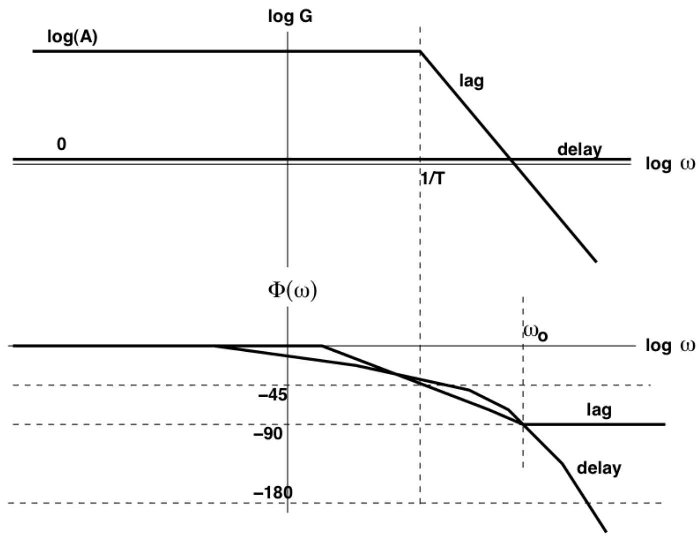

Figure 5.26 *Bode plot for the two subsystems.*

Problem 5.6 The wave equation of the BM as deduced by Von Békésy and Zwislocki is linear, which means that the superposition principle should hold, and that the output amplitude of the BM is independent of stimulus amplitude. However, when you listen (carefully) to a superposition of two frequencies, say f_1 = 440 Hz (musical 'A') and f_2 = 523 Hz (musical 'C'), you can hear the presence of a third tone with a frequency that is close to the musical 'F' (349 Hz)! This additional tone is a *combination tone*, and results to have a frequency of $2f_1$-f_2 = 357 Hz. This combination tone is a manifestation of a nonlinearity in the system, and is due to the nonlinear cochlea (note that the effect disappears when the two tones are presented to different ears!). Suppose, for simplicity, that the output of the BM, *q(t)*, depends on the instantaneous pressure, *p(t)*, through the following nonlinear third-order relation (a third-order Taylor approximation on the nonlinear transfer function):

$$q(t) = a \cdot p(t) + b \cdot p^2(t) + c \cdot p^3(t) \qquad (5.68)$$

(a) By presenting a superposition of two harmonic waves at the input, say

$$p(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$$

show that the output of the system can described by a spectrum that contains 13 frequencies! Determine also their relative amplitudes. Which nonlinear term causes the observed 'F' percept?

(b) What happens to the amplitude(s) of the distortion products if the input is given by

$$p(t) = A\cos(\omega_1 t)$$

Problem 5.7 Analysis of a Hopf bifurcation. Consider the Hopf bifurcation in polar coordinates (with a and μ non-zero real-valued parameters):

$$\frac{dr}{dt} = r \cdot (a - \mu \cdot r^2)$$
$$\frac{d\phi}{dt} = \omega_0 \qquad (5.69)$$

Investigate the stability of the fixed points as function of the parameters. When do we see a limit cycle? What determines its amplitude and frequency?